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NUMERICAL 3-D SEISMIC MODELLING

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Summary

The objective of this research project is to develop and test algorithms and computer programs that enable a realistic modelling of seismic wave propagation in 3-D (three-dimensional) laterally inhomogeneous media. Various material rheologies are considered: acoustic/elastic, viscoacoustic/-elastic, anisotropic. The numerical solution of the equations of motion is obtained by the Fourier method. New time integration schemes are developed which are superior to the conventional finite difference time stepping approach.

Time sections and snapshots of the wavefield for 2-D and 3-D laterally inhomogeneous media of different rheologies are presented.

1. INTRODUCTION

The growing importance of 3-D seismic surveys in hydrocarbon exploration requires the development of appropriate forward modelling techniques for quality control and interpretation of results.

Appropriate in our opinion is a wide-band technique suitable for the simulation of the full wave response in 3-D generally varying media and allowing the implementation of various rheologies.

Numerical methods which are based on finite difference techniques cannot be applied to 3-D exploration models of realistic size and rheology because they require a computing capacity which cannot be met even by the supercomputers available today.

The proposed approach is based on the Fourier method [1]. The main advantage of this method over finite difference techniques is that the ratio of minimum wave length to the spatial increment is three times smaller. This permits a significant reduction of the working space dimensions so that a seismic modelling of realistic 3-D structures can be performed. Computation time however remains large, in particular when finite difference time stepping is used.

In order to improve this situation, we have developed two new techniques of time integration in this project which do not require a restrictive stability criterion. Wave field calculations by these methods are either more accurate [2] or more economical [3] than the conventional time stepping approach.

Based on these numerical methods, computer programs have been developed for 2-D and 3-D acoustic and and elastic modelling. For the viscoacoustic and -elastic case 2-D FORTRAN programs have been completed, while those for the 3-D case are being developed. Our work on the modelling of wave propagation in anisotropic media has been focussed so far on the transverse-isotropic case.

The accuracy of the algorithms has been tested for simple models by comparison with analytical solutions obtained by Cagniard's method and showed excellent agreement.

2. 3-D-MODELLING BY THE FOURIER METHOD

Direct methods for the numerical solution of the equations of motion in inhomogeneous media require spatial derivatives of the wave field. These can be calculated accurately by the use of Fourier transforms as described in [1]. This method is outlined here for the 3-D acoustic case.

The 3-D acoustic wave equation with variable density can be expressed as

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right)=\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2}+S, \tag{2.1}$$

where $p(x,y,z,t)$ denotes the pressure, $\rho(x,y,z)$ the density, $c(x,y,z)$ the P-wave velocity and $S(x,y,z,t)$ the source term which vanishes outside a region near the source location.

Temporal and spatial discretization of eq. (2.1) leads to the finite difference equation

$$L^n_{ijk}=\frac{1}{\rho_{ijk}c^2_{ijk}}\left[\dot{p}^{n+\frac{1}{2}}_{ijk}-\dot{p}^{n-\frac{1}{2}}_{ijk}\right]+S^n_{ijk}, \tag{2.2}$$

where $\dot{p}^{n+\frac{1}{2}}_{ijk}$ and $\dot{p}^{n-\frac{1}{2}}_{ijk}$ denote the temporal derivatives of the pressure at times $t=(n+\frac{1}{2})\Delta t$ and $t=(n-\frac{1}{2})\Delta t$, respectively, and where S^n_{ijk} denotes the source term at time $t=n\Delta t$ at the point $((i-1)\Delta x,(j-1)\Delta y,(k-1)\Delta z)$. L^n_{ijk} denotes the discrete approximation of the left hand side of eq. (2.1).

As initial conditions corresponding to time $t = 0$ and $t = -\frac{1}{2}\Delta t$, respectively, the variables p and $\dot{p}$ are set to zero. For each time step the expressions

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right), \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right), \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) \tag{2.3}$$

are calculated by use of the fast Fourier transform (FFT) and its inverse FFT^{-1}, respectively, according to the following scheme:

$$p \xrightarrow{\mathrm{FFT}} \hat{p} \xrightarrow{\cdot ik_x} ik_x\hat{p} \xrightarrow{\mathrm{FFT}^{-1}} \frac{\partial}{\partial x}p \xrightarrow{\cdot\frac{1}{\rho}} \frac{1}{\rho}\frac{\partial p}{\partial x} \xrightarrow{\mathrm{FFT}} \widehat{\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)}$$

$$\xrightarrow{\cdot ik_x} ik_x\widehat{\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)} \xrightarrow{\mathrm{FFT}^{-1}} \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)$$

Having calculated three terms in (2.3) for all $N_xN_yN_z$ grid points the time integration can be performed as follows:

$$\begin{aligned} \dot{p}_{ijk}^{n+\frac{1}{2}} &= \dot{p}_{ijk}^{n-\frac{1}{2}} + \rho_{ijk}c_{ijk}^2\Delta t\left[L_{ijk}^n - S_{ijk}^n\right] \\ p_{ijk}^{n+1} &= p_{ijk}^n + \Delta t\dot{p}_{ijk}^{n+\frac{1}{2}} \end{aligned} \tag{2.4}$$

3. IMPROVED TECHNIQUES OF TIME INTEGRATION

Based on a work of Tal Ezer [3] a new time integration technique has been developed. This method starts from the wave equation expressed as a system of first order partial differential equations.

Instead of approximating the formal solution by a Taylor series which leads to finite difference schemes, a Chebychev expansion is used. This yields a solution at arbitrary time steps, without restrictions from a stability criterion. As this technique avoids numerical dispersion, it is particulary well suited for viscoelastic modelling, where physical dispersion occurs. However, this method requires about the same computational effort as the usual temporal differencing schemes.

Using a different operator formulation of the wave equation, we developed the so-called rapid expansion method (REM) [2]. This results in comparable accuracy with only half of the computational effort. For the sake of simplicity this method will now be described for the 1-D acoustic case.

The 1-D acoustic wave equation can be written in the compact form

$$\frac{\partial^2 p}{\partial t^2} = -L^2 p + S(x)h(t), \tag{3.1}$$

where $L^2 = -c^2(x)\frac{\partial^2}{\partial x^2}$. Discretizing at the N_x points $x_j = j\Delta x$, $j = 0, 1, \ldots, N_x - 1$ and approximating the spatial derivatives by the Fourier method leads to the system of N_x ordinary differential equations

$$\ddot{\underline{p}} = \hat{L}^2 \underline{p}(t) + h(t)\underline{S} \tag{3.2}$$

where $\underline{p}(t)$and $\underline{S}$ are given by

$$\underline{p}(t) = [p(0,t), p(\Delta x, t), \ldots, p((N_x - 1)\Delta x, t)]^T,$$

$$\underline{S} = [S(0), S(\Delta x), \ldots, S((N_x - 1)\Delta x)]^T.$$

$\hat{L}^2$ denotes the approximation of L^2 by the Fourier method. The formal solution of this system is

$$\underline{p}(t) = \cos \hat{L}t \quad \underline{p}(0) + \frac{\sin \hat{L}t}{\hat{L}} \dot{\underline{p}}(0) \tag{3.2a}$$

for a vanishing source term and

$$\underline{p}(t) = \left(\int_0^t \frac{\sin \hat{L}\tau}{\hat{L}} h(t - \tau) d\tau \right) \underline{S} \tag{3.2b}$$

for a non-zero source term with vanishing initial conditions. Adding the expressions for $\underline{p}(t)$ and $\underline{p}(-t)$ from (3.2a) gives

$$\underline{p}(t) = -\underline{p}(-t) + 2\cos \hat{L}t \quad \underline{p}(0), \tag{3.3}$$

where the operator $\cos \hat{L}t$ can be expressed as

$$\cos \hat{L}t = \sum_{k=0}^{\infty} c_{2k} J_{2k}(Rt) Q_{2k}\left(\frac{i\hat{L}}{R}\right). \tag{3.4}$$

For a non-zero source term we use the expansion

$$\frac{\sin \hat{L}\tau}{\hat{L}} = \sum_{k=0}^{\infty} c_{2k+1} \frac{J_{2k+1}(\tau R)}{R} \frac{R}{i\hat{L}} Q_{2k+1}\left(\frac{i\hat{L}}{R}\right). \tag{3.5}$$

Thus one obtains the expression

$$\underline{p}(t) = \sum_{k=0}^{\infty} \left(\int_0^t \frac{c_{2k+1}}{R} J_{2k+1}(R\tau) h(t - \tau) d\tau \right) \frac{R}{i\hat{L}} Q_{2k+1}\left(\frac{R}{i\hat{L}}\right) \underline{S}. \tag{3.6}$$

Since there are only even or odd indices in eqs. (3.4) and (3.5), respectively, the computational effort of the REM is only half of the effort of the method described in [3].

4. HYBRID MODELLING

Based on [4] und [5] a new hybrid modelling method is in development for a calculation of the wave field at irregular interfaces by means of local boundary Green's function. This approach is expected to perform a forward modelling very efficiently in particular for partially homogeneous media.

5. ABSORBING BOUNDARIES

The use of the discrete Fourier transform implies a periodic continuation of the model in each spatial direction.

Usually, this periodicity causes waves incident at a model boundary, to be folded back onto the opposite boundary and to be propagated again through the model. This cascading of propagation by wraparound must be suppressed. This is done by introducing absorbing model boundaries.

The absorbing boundary zone usually covers a range of 15 grid points at each of the model boundaries. Damping in this zone is achieved by multiplying the pressure p and its time derivative $\dot{p}$ by a weighting function w (with $|w| < 1$).
This absorbing boundary zone corresponds to a modified wave equation published in [6], [7].

6. NECESSARY COMPUTER RESOURCES

Seismic 3D modelling requires extensive computer resources. Memory size and computing time will be determined by the following criteria:

Nyquist Condition

The spatial increment $\Delta x = \Delta y = \Delta z$ must be chosen according to the relation:

$\Delta x \leq \frac{1}{2}\lambda_{min}$,

where λ_{min} denotes the Nyquist wavelength.

Stability Condition

The time increment Δt is subjected to the equation

$\alpha = \frac{c_{max}}{\Delta x}\Delta t < \frac{2}{\pi\sqrt{3}}$

where c_{max} denotes the maximum velocity of propagation in the model.

This stability condition is not required when using the improved time integration methods described in chapter 3.

Storage Requirements

Let N_x, N_y and N_z denote the number of grid points in the x, y and z directions. The numerical solution of eq. (2.1) requires four global arrays of size $N_x N_y N_z$ for the variables p, $\dot{P}$, ρ and c. One auxiliary array of the same size is required for accumulating the terms on the left-hand side of eq. (2.1). Consequently for the 3-D elastic case the storage requirements are three times larger.

A typical elastic problem as outlined in Fig. 3 requires approx. 95 megawords of storage.

7. VISCOACOUSTIC/-ELASTIC MODELLING

Wave propagation in the earth has always been recognized as anelastic. Simulations which attempt an accurate amplitude reconstruction must therefore be able to account for the effects of attenuation and dispersion.

In a first stage we solve the equations of motion which refer to the propagation of a compressional wave field including anelastic effects. The theory presented in detail in [8] implies viscoacoustic wave propagation in one-, two- and three-dimensional earth materials. The modelling program has been implemented to solve the 2-D wave propagation problem, the corresponding 3-D program is in development. The model uses a spectrum of relaxation mechanisms to describe the relation between strain and stress. The problem posed by the convolutional integral is circumvented by the introduction of memory variables. The modelling code was verified by comparison with analytic solutions for wave propagation in a homogeneous medium.

The next stage we developed the viscoelastic wave propagation theory [9]. This type of description yields more accurate results than the viscoacoustic for the determination of the wave amplitudes, and gives a distinction between P and S waves. The theory is based on the standard linear solid rheology and uses a spectrum of relaxation mechanisms to describe the anelastic effects acting on the compressional and shear wave fields. The model is solved for the three dimensions of the space.

8. NUMERICAL EXAMPLES

Four simple models have been chosen to demonstrate the present status of the project. They are shown in Fig. 1 to 4 on the following pages.

NUMERICAL 3D-SEISMIC MODELLING

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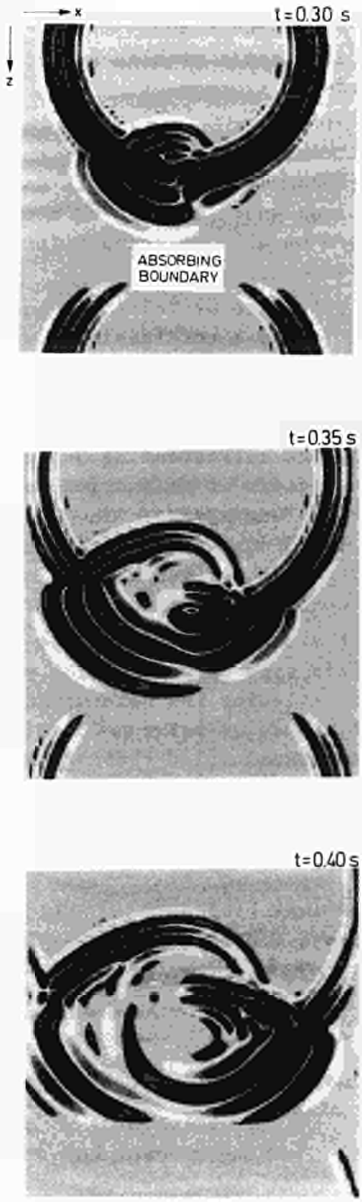
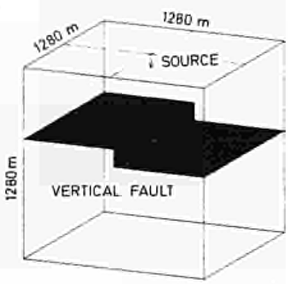
SNAPSHOTS → OF PRESSURE WAVEFIELD





SPECIFICATIONS : FOURIER - METHOD

P-WAVE VELOCITY

UPPER	MEDIUM	2000 m/s
LOWER		4000 m/s

CONSTANT DENSITY
EXPLOSIVE POINT SOURCE
RICKER WAVELET (25 Hz)
BAND WIDTH 0-50 Hz
64 × 64 × 64 GRID ELEMENTS
GRID SPACING 20 m

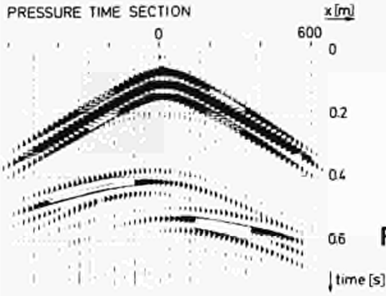


Fig. 1

NUMERICAL 3D-SEISMIC MODELLING

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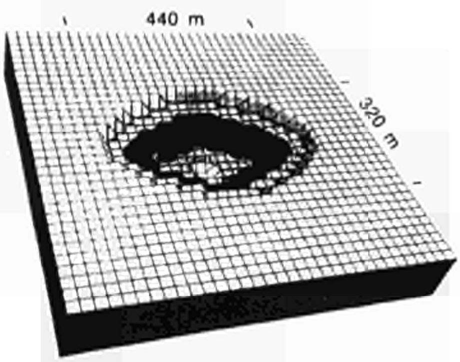
3D - ACOUSTIC MODEL



Pressure Snapshot t=300 ms

SPECIFICATIONS: FOURIER-METHOD

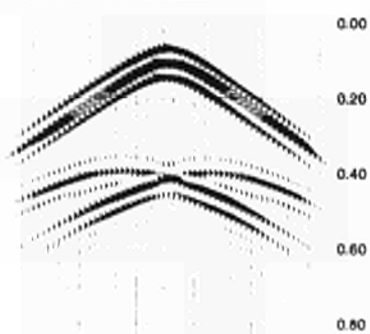
P-WAVE VELOCITY:

UPPER	MEDIUM:	2000 m/s
LOWER		4000 m/s

CONSTANT DENSITY
EXPLOSIVE POINT SOURCE
RICKER WAVELET (25 Hz)
BAND WIDTH 0 . . . 50 Hz
64 x 64 x 64 GRID ELEMENTS
GRID SPACING 20 m

Pressure Snapshot t=350 ms

Pressure Time Section



Pressure Snapshot t=400 ms

Fig. 2

NUMERICAL 3D SEISMIC MODELLING

(UNIVERSITY OF HAMBURG)

3D ELASTIC SYNCLINE MODEL

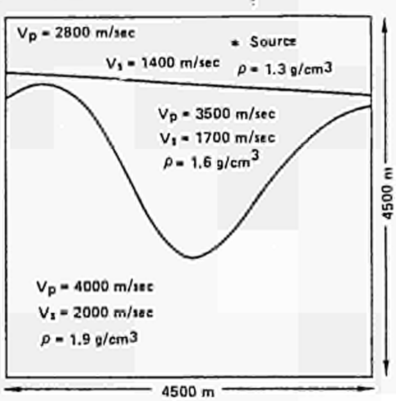
EXPLOSIVE POINT SOURCE
RICKER WAVELET
BAND WIDTH 0-35 Hz

FOURIER METHOD

SPATIAL GRID
NUMBER OF GRID POINTS
Nx x Ny x Nz = 225 x 125 x 225
GRID SPACING 20m
1500 TIME STEPS

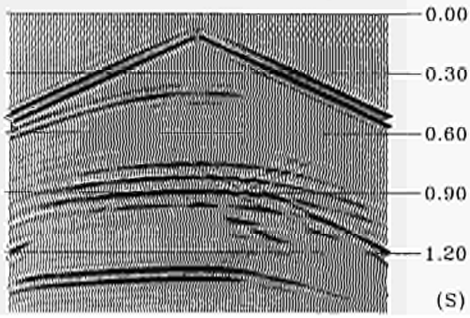
TOTAL COMPUTATION TIME 4.5 HOURS ON CRAY X-MP/48 + SSD

XZ PLANE

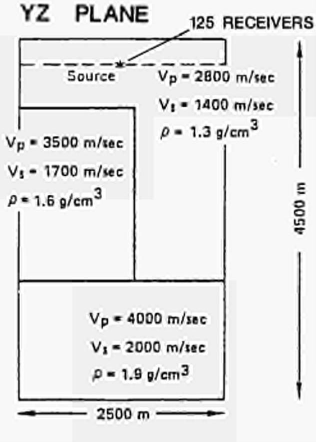


SINGLE SHOT TIME SECTIONS

PRESSURE



YZ PLANE



VERTICAL (Z) DISPLACEMENT

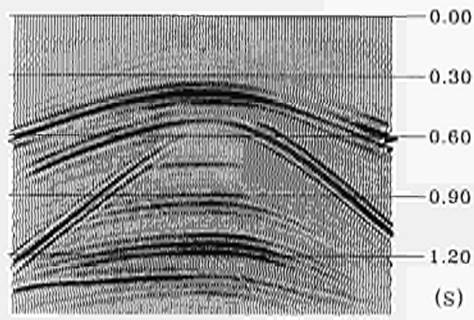


Fig. 3

VISCOELASTIC SEISMIC MODELING

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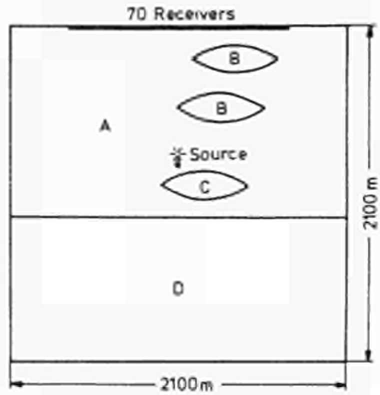


TWO-DIMENSIONAL MEDIUM

RHEOLOGY

GENERALIZED STANDARD LINEAR SOLID
CONSTANT-Q MEDIA USING TWO SETS OF RELAXATION MECHANISMS FOR EACH PROPAGATING MODE

TIME INTEGRATION

BEST POLYNOMIAL APPROXIMATION OF THE EVOLUTION OPERATOR IN THE COMPLEX PLANE

SPATIAL DERIVATIVES

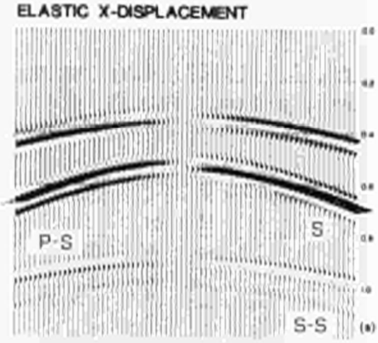
FOURIER METHOD
NUMBER OF GRID POINTS
Nx x Ny = 105 x 105
GRID SPACING 20m

VERTICAL POINT SOURCE
RICKER WAVELET
PEAK FREQUENCY 20 Hz

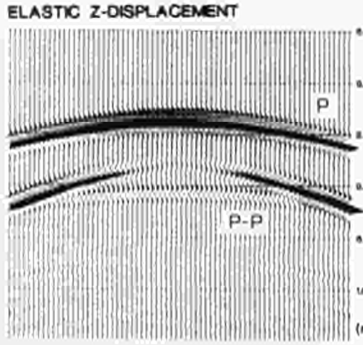
MATERIAL	Vp (20 Hz) (m/s)	Vs (20 Hz) (m/s)	DENSITY (g/cm3)	Qp	Qs
A	2800	1700	2	145	100
B	2800	1700	2	15	10
C	2800	1700	2	30	20
D	4000	2500	2	∞	∞

SINGLE SHOT TIME SECTIONS

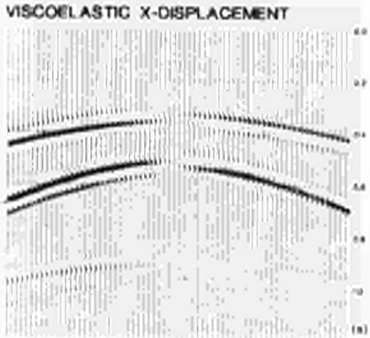
ELASTIC X-DISPLACEMENT



ELASTIC Z-DISPLACEMENT



VISCOELASTIC X-DISPLACEMENT



VISCOELASTIC Z-DISPLACEMENT

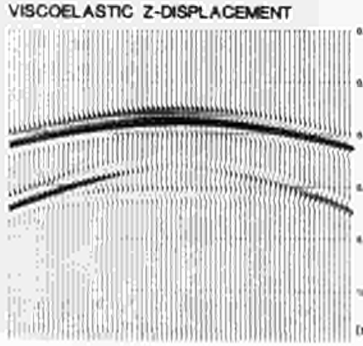


Fig. 4

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