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Simulation of ultrasonic waves in a natural sandstone *

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Abstract

We use Biot's theory to model microseismograms obtained at ultrasonic frequencies for a clean, clay-free sandstone. A qualitative match between the synthetic and experimental traces is obtained by using a viscoelastic formulation of the theory. The modeling of the observed strong attenuation of the slow and shear waves requires the generalization of the Biot moduli and viscosity/permeability factor to time-dependent relaxation functions. The modeling algorithm is parameterized with the dry-rock bulk and rigidity moduli and the tortuosity to match the observed travel-times and with the quality factors associated to the relaxation functions to match the observed amplitudes.

1 Introduction

Acoustic experiments in the laboratory can be used to verify and calibrate a suitable theo for computing synthetic seismograms. The experiment conducted by Kelder and Smeuld [6] provides such a basis for theory and numerical-algorithm verification. They obta transmission microseismograms through a slab made of a clean and unconsolidated natur sandstone, where all the events predicted by Biot's theory can be observed. The experime provides, under controlled conditions, the travel times and relative amplitudes of the fa and slow compressional waves and the shear wave (Figure 1).

We model the microseismograms by using Biot's theory, introducing stiffness a viscodynamic dissipation based on viscoelastic theory, to model additional attenuati mechanisms. This approach was proposed by Carcione [3] to model relaxation mechanism arising from the interaction between the skeleton and the porefluid. In that case, t Biot solid-grain/porefluid coupling modulus is generalized to a time-dependent relaxatio function. Here, we show that the experimental attenuation levels observed by Kelder an Smeulder [6] can be modeled by making viscoelastic the rigidity modulus and the flu viscosity/permeability factor, in addition to the above mentioned coupling modulus.

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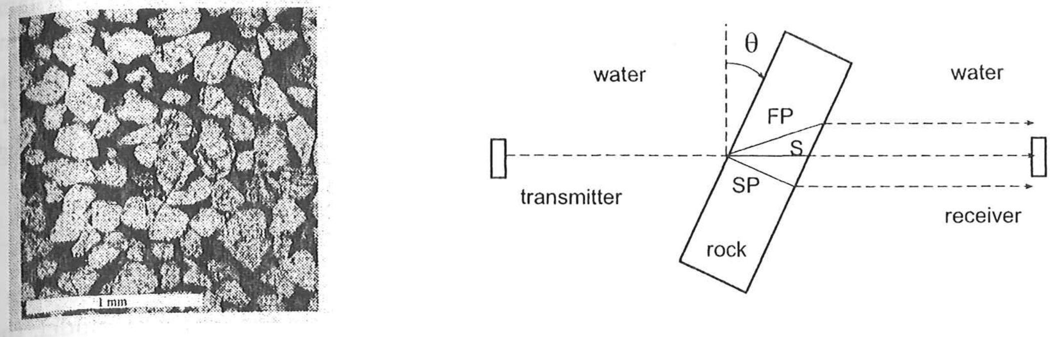

FIG. 1. Left: *photomicrograph of the Nivelsteiner sandstone sample (after Kelder and Smeulders [6]).* Right: *experimental setup, where FP denotes the fast compressional wave, SP denotes the slow compressional wave and S denotes the shear wave.*

2 Poroelastic equations of motion

The differential equations for an inhomogeneous, isotropic poroelastic medium under plane strain conditions are given by [1, 3]

$$\begin{aligned}
\tau_{xx,t} &= E v_{x,x} + (E - 2\mu) v_{z,z} + \alpha M \epsilon + s_x, \\
\tau_{zz,t} &= (E - 2\mu) v_{x,x} + E v_{z,z} + \alpha M \epsilon + s_z, \\
\tau_{xz,t} &= \mu (v_{x,z} + v_{z,x}) + s_{xz}, \\
p_{,t} &= -M\epsilon + s_f, \\
\epsilon &= \alpha (v_{x,x} + v_{z,z}) + q_{x,x} + q_{z,z}, \\
\tau_{xx,x} + \tau_{xz,z} &= \rho v_{x,t} + \rho_f q_{x,t}, \\
\tau_{xz,x} + \tau_{zz,z} &= \rho v_{z,t} + \rho_f q_{z,t}, \\
-p_{,x} &= \rho_f v_{x,t} + m q_{x,t} + \frac{\eta}{\kappa} q_x, \\
-p_{,z} &= \rho_f v_{z,t} + m q_{z,t} + \frac{\eta}{\kappa} q_z,
\end{aligned} \tag{1}$$

where the first five equations are the stress/particle-velocity relations, the next two equations are Biot/Newton's differential equations, and the last two equations are dynamic Darcy's law; τ_{xx}, τ_{zz} and τ_{xz} are the total stress components, p is the fluid pressure, the v's and the q's are the solid and fluid (relative to the solid) particle velocities, and s_x, s_z, s_{xz} and s_f are the external sources of stress, for the solid and the fluid, respectively. The composite density is

$$\rho = (1 - \phi)\rho_s + \phi \rho_f$$

with ρ_s and ρ_f the solid and fluid densities, respectively. Moreover, $m = T\rho_f/\phi$ with T the tortuosity, η is the fluid viscosity, and κ is the permeability of the medium. Finally, the elastic coefficients are given by

$$E = K_m + \frac{4}{3}\mu, \quad M = \frac{K_s^2}{D - K_m}, \quad D = K_s[1 + \phi(K_s K_f^{-1} - 1)], \quad \alpha = 1 - \frac{K_m}{K_s},$$

with K_m, K_s and K_f the bulk moduli of the drained matrix, solid and fluid, respectively; ϕ is the porosity, and μ is the shear modulus of the drained (and saturated) matrix. The subscript "$,x$" denotes $\partial/\partial x$.

3 Extension to the poroviscoelastic case

Viscoelasticity is introduced into Biot's poroelastic equations for modeling attenuation related to the potential energy (stiffness dissipation) and the kinetic energy (viscodynamic dissipation) [1, 2]. In the first case, the stiffnesses E, μ and α are generalized to time-dependent relaxation functions, which we denote, in general, by $\psi(t)$. We assume that $\psi(0) = \psi_0$ equals the respective Biot modulus, i.e., we obtain Biot's poroelastic constitutive equations at high frequencies. Assume that the relaxation functions are described by a single Zener model,

$$\psi(t) = \psi_0 \frac{\tau_\sigma}{\tau_\epsilon}\left[1 + \left(\frac{\tau_\epsilon}{\tau_\sigma} - 1\right)\exp(-t/\tau_\sigma)\right] H(t), \tag{2}$$

where $H(t)$ is the Heaviside function, and τ_ϵ and τ_σ are relaxation times. In the absence of experimental values for attenuation versus frequency we consider the simplest model, that is, a single relaxation peak for each modulus with peak frequency equal to the frequency of the transducers. Viscoelasticity implies that multiplications of bulk moduli by field variables in equations (1) be replaced by time convolutions. For instance, in the first equation these products are $E(v_{x,x} + v_{z,z})$, $\mu v_{z,z}$ and $\alpha\epsilon$. We substitute them by $\psi * u_{,t}$, where ψ denotes the relaxation function corresponding to E, μ or M, u denotes $v_{x,x} + v_{z,z}$, $\mu v_{z,z}$ or $\alpha\epsilon$, and $*$ indicates time convolution. As in the single-phase viscoelastic case, we introduce memory variables to avoid the time convolutions. Then, the terms $\psi * u_{,t}$ are replaced by $\psi_0 u + e$, where e is the memory variable. There are five memory variables related to the constitutive equations, which satisfy the following differential equation:

$$e_{,t} = \psi_0\left(\frac{1}{\tau_\epsilon} - \frac{1}{\tau_\sigma}\right) u - \frac{e}{\tau_\sigma}. \tag{3}$$

On the other hand, viscodynamic dissipation introduces two additional memory variables through the generalization of $b = \eta/\kappa$ in equations (1) to a time-dependent viscodynamic operator $b(t)$. In this case, Darcy's equations are obtained at the low frequency limit, i.e., for $t \to \infty$ [2].

The relaxation times can be expressed in terms of a Q factor Q_0 and a reference frequency f_0 as

$$\tau_\epsilon = (2\pi f_0 Q_0)^{-1}[\sqrt{Q_0^2 + 1} + 1], \quad \text{and} \quad \tau_\sigma = \tau_\epsilon - (\pi f_0 Q_0)^{-1}. \tag{4}$$

4 Numerical algorithm

The use of a staggered mesh greatly improves the accuracy of the modeling algorithm [7, 4]. On a regular grid the field components and material properties are represented at each grid point, while on a staggered grid, variables and material properties are also defined at half-grid points. Material properties at half-grid points are computed by averaging the values defined at regular points [4]. The first-order derivative computed with the staggered differential operator is evaluated between grid points and uses even-based Fourier transforms. The spatial derivatives are calculated with the fast Fourier transform (FFT). This approximation is infinitely accurate for band-limited periodic functions with cutoff spatial wavenumbers which are smaller than the cutoff wavenumbers of the mesh. Since the presence of the slow compressional wave makes Biot's differential equations stiff, a time-splitting integration algorithm is used [5]. The stiff part is solved analytically and the non-stiff part with an A-stable Crank-Nicolson scheme. This method possesses the stability properties of implicit algorithms, but the solution can be obtained explicitly. It

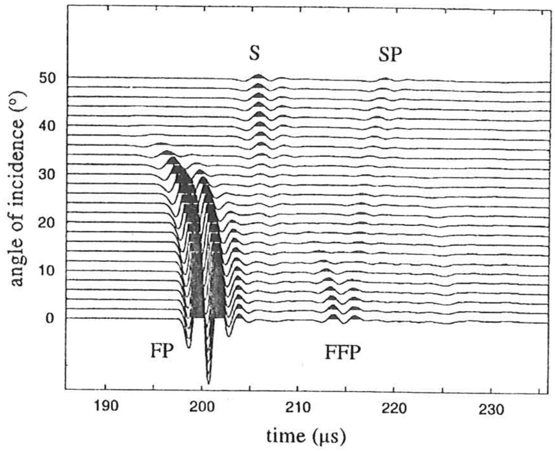

FIG. 2. *Microseismogram obtained by Kelder and Smeulders [6] for Nivelsteiner sandstone as a function of the angle of incidence θ. The events are the fast compressional wave (*FP*), the shear wave (*S*), the first multiple reflection of the fast compressional wave (*FFP*) and the slow wave (*SP*).*

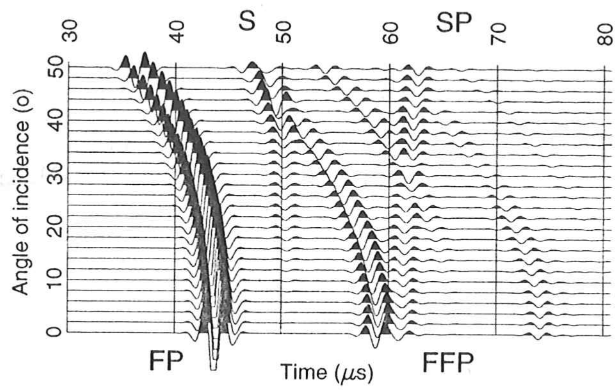

FIG. 3. *Numerical microseismogram corresponding to the experiment of Kelder and Smeulders [6], obtained from Biot's poroviscoelastic theory. The dominant frequency of the source is 500 kHz.*

can be shown that the splitting technique and the Crank-Nicolson scheme have 2nd-order accuracy.

5 Simulation

The Nivelsteiner-sandstone sample used by Kelder and Smeulders [6] in their experiment is a Miocene quartz sand with very low clay content. It has an average grain distribution of 100-300 μm (Figure 1). For simplicity we assume that the grains are made of pure quartz. We obtain the matrix properties by fitting the experimental data provided by Kelder and Smeulders at 500 kHz, and assuming that the level of dissipation is that predicted by Biot's theory. Firstly, we compute the shear modulus of the dry rock, μ, by fitting the shear-wave velocity of the saturated rock. Secondly, the dry-rock bulk modulus, K_m, is obtained by fitting the compressional velocity of the saturated rock. Finally, the tortuosity T is used as a free parameter to fit the experimental slow-wave velocity.

The experimental setup is shown in Figure 1. A sample of Nivelsteiner sandstone 21 mm thick, immersed in water, is mounted on a rotating table. The traces are obtained for several values of the angle θ. The experimental results are shown in Figure 2, where the traces are plotted versus the angle of incidence θ. A 2D cross section of this model is discretized on a mesh with 238 $\times$ 238 grid points and a grid spacing $D_X = D_Z = 0.5$ mm. Water is modeled by setting $K_m = \mu = 0$, $K_s = K_f$ and $\rho_s = \rho_f$. The source is a Ricker-type wavelet and has a central frequency of 500 kHz, and the algorithm uses a time step of 0.03 μs and 3340 steps.

To model the same level of relative attenuation between the different events, we generalize μ, M and b to relaxation functions, with $Q_0^{\mu} = 10$, $Q_0^M = 10$ and $Q_0^b = 2$, respectively, and a relaxation-peak frequency of 250 kHz. Figure 3 shows the poroviscoelastic microseismograms. As can be appreciated, the relative amplitudes observed in Figure 3 are in good agreement with the experiment.

6 Conclusions

The observed wave attenuation in reservoir rocks can be due to several dissipation mechanisms. Macroscopic and local fluid-flow mechanisms (Biot and squirt flow, respectively) are two of them and, in principle, can be used to fit experimental data. However, these mechanisms are not enough to model the observed levels of attenuation. Moreover, the strong dependence of these models on the microstructural features makes them not very reliable for attenuation prediction. A practical model, which requires calibration with controlled acoustic experiments, is the viscoelastic model parameterized by reference frequencies and quality factors associated to the stiffness-moduli and the viscosity/permeability factor. Laboratory measurements of wave velocity and attenuation factor versus frequency provide the tool for obtaining these empirical parameters and computing realistic synthetic seismograms.

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