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Short Note

Radiation patterns: Possibility for monitoring seismic sources

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INTRODUCTION

Recent field measurements of the radiation in boreholes indicate that the radiation patterns of real seismic sources are not always in agreement with those corresponding to the point-source excitation in unbounded homogeneous and isotropic acoustic or elastic media [we refer the reader to Aki and Richards (1980) for the basic discussion on the radiation patterns in homogeneous media]. This mismatch results from the fact that the point-source radiation patterns corresponding to homogeneous media are too simplistic to satisfy any experiment in the more realistic Earth environment. A study of radiation patterns is certainly important not only to predict possible seismic events but also to analyze the source performance itself by recording seismic arrivals.

In this paper, we will study the radiation patterns of vertical and rotational loads applied to a disk-shaped surface and will demonstrate an intriguing possibility of monitoring seismic sources by measuring the three-component displacement vector at the Earth's surface. The study has several practical applications: for example, if a source of seismic energy corresponds to a rotational load (as in a drill-bit experiment), by measuring the three-component displacement vector, one can evaluate whether the drilling performance of the source is satisfactory or not. In fact, in a drilling experiment, the drill-bit teeth generate a combination of waves that propagate into the Earth and along the drill string. The radiation component that travels into the Earth is also a combination of both axial and tangential torque load excitations.

We show that by using the computed radiation patterns, one can monitor the drill bit in time and draw a conclusion about the impact of a bit. For example, the action of a roller-cone drill bit is characterized mainly by vertical force, whereas a diamond bit crushes rocks with grinding horizontal forces. Knowing the radiation patterns of both types of drill bits enables one to determine at which times the grinding efficiency of the diamond drill bit is small and when the roller-cone drill bits are more or less efficient.

The computed radiation patterns were calculated for two extreme cases: the first one corresponds to large wavelengths (compared to the layer thicknesses). In this case, the Earth acts as a homogeneous anisotropic transversely isotropic (TI) medium affecting the source radiation pattern. In the second case, the wavelength is comparable to the thicknesses of the layers. As a consequence, the waves are characterized by ringing coda. These radiation patterns predict the performance of sources in the Earth.

LONG WAVELENGTH RADIATION OF DRILL

Vertical loads

Studies of the radiation patterns of a monopolar point source acting along the borehole goes back to White (1965) and Heelan (1953). They found the analytical expressions for far-field P and SV radiations. Since their studies conform to a homogeneous and isotropic medium, more realistic developments are necessary. The latest work of Rector and Hardage (1992) also treats the radiation patterns of drill-bit sources using an assumption of a homogeneous, isotropic medium. In this case, the radiation can be described by the analytical expressions.

We will show how a more realistic assumption (as the locally layered Earth) significantly changes the computed radiation patterns. We will start with a vertical force

$$\mathbf{f} = (0, 0, W(t))\delta(\mathbf{x} - \mathbf{x}_s), \tag{1}$$

where $W(t)$ is the source wavelet, $\mathbf{x} = (x, y, z)^T$ is the coordinate vector, and $\mathbf{x}_s \in R^2$ is the source coordinate. The corresponding equations of motion are

$$\rho\mathbf{u} = \nabla \cdot \Sigma + \mathbf{f}, \tag{2}$$

where $\mathbf{u}$ is the displacement vector and Σ is the stress tensor. In the case of long wavelengths, one can use the Backus (1962) averaging technique to replace a layered medium by an effective TI medium. A constitutive equation of a 3-D TI medium is given by

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$$\begin{vmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{vmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} \begin{vmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{vmatrix}, \tag{3}$$

where σ_{ij} and ε_{ij} are the components of the stress and strain tensors. The effective elastic moduli are described by the Lamé constants [see for example, Carcione and Carrion (1992)].

For numerical simulations, we choose the following model: layers of sandstones and limestones 5-m thick with the compressional velocities 2950 m/s and 5440 m/s, respectively, are mixed so the total number of sandstone and limestone layers remains the same (stationarity). As mentioned above, if the wavelength of the propagating waves is much greater than the layer thicknesses, the medium acts as a homogeneous transversely isotropic (TI) medium (Backus, 1962; Schoenberg and Muir, 1989.) Transverse isotropy (TI) means that in the radial direction the medium is isotropic. In fact, the cut-off frequency was taken as 60 Hz, which corresponds to wavelengths of the order 50–100 m. For these wavelengths, layer thicknesses of the order 1–10 m can be considered small. Let us study the radiation pattern of the vertical load distributed over a disk of small radius placed in the 3-D medium. Carrion and Sampaio (1991) considered a similar problem and found an analytical solution for a disk-type source and an arbitrary distributed load over the disk. Carcione and Carrion (1992) performed numerical simulations in a 3-D anisotropic medium and found that anisotropy can significantly change the radiation patterns of point sources (Schoenberg, 1986). It is possible to prove that the distribution of a load over the disk does not play any significant role provided the radius is much smaller than the characteristic wavelength of the source.

In the numerical modeling, we have used the time integration scheme rapid expansion method (REM) to solve the wave equation. Spatial derivatives are calculated by using the Fourier pseudospectral method based on the prime factor FFT. In this case, the length of each FFT is a product of odd prime numbers. It is important to emphasize that the present method uses both spectral methods in time and in space and thus avoids any type of numerical dispersion.

Let us turn to Figure 1 that displays the 3-D radiation pattern of the vertical load in the Cartesian coordinate system. (Z-axis is always vertical; in the XY-plane, Y-axis is always positive.) We used a spectral modeling technique that is known to yield no numerical dispersion up to the Nyquist frequency of the mesh (Carcione et al., 1992). The top row represents all three components of the displacement field measured at the vertical XZ-plane that passes through the center of the disk. One can see that the wavefront of the quasi-compressional wave is not spherical with lobes at the bisects of the quadrants (x-component), and the quasi-shear (x-component of the SV-wave) is gathered into cusps. The YZ-plane demonstrates symmetry with the XZ-plane (second row). In this case, the z-component will remain the same (compared to the top row), whereas the y-component will be replaced by the x-component of the displacement at the XZ-plane. The figures in the third row show the displacement components in the horizontal plane passing through the source location [the only nonzero component represents the vertical displacement which is a combination of the compressional and shear (qSV) waves]. It is interesting to observe two events. First, there is a strong coupling between the quasi-compressional and quasi-shear waves caused by anisotropy, but there is no coupling between compressional and shear waves in the XY plane where the medium acts as homogeneous and isotropic. Second, one can notice the near-field effects seen distinctly in all three rows.

Rotational load

We will now consider the same 3-D medium but instead of the vertical load, we will treat a rotational load distributed over the disk. The equations of motion can be written in the following manner:

$$\rho \mathbf{u} = \nabla \cdot \Sigma + \mathbf{F}, \tag{4}$$

where $\mathbf{F}$ is expressed with respect to the vector potential A:

$$\mathbf{F} = \left(\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}, 0 \right), \tag{5}$$

where

$$A = \psi(\mathbf{x}) W(t) \hat{e}_z, \tag{6}$$

and $\hat{e}$ is the unit vector. Here ψ is a Gaussian distribution function with the maximum at the center of the disk. Let us consider Figure 2. One can see that the wavefronts are not spherical. In the case of the rotational load, the only nonzero component refers to the SH-wave. The first row of Figure 2 is the recorder displacement field at the XZ vertical plane, whereas the second row shows the displacement field in the YZ vertical plane. In the horizontal plane, the displacement field is represented by the x and y nonzero components of the displacement. Using these results, one can compute the angular component of the displacement which possesses the axial symmetry. The computed radiation patterns can serve as a tool for the monitoring of seismic sources. Suppose, for example, that a source placed in a borehole is a combination of the vertical and the rotational loads. By measuring the y-component of the displacement in the XZ plane, one can immediately see when the source works as a rotational load and when it does not. If, for example, the y-directed geophone shows relatively large amplitudes away from the zero offset, the source operates in a grinding mode. If the vertical displacement is quite large at the surface of Earth (in the XZ plane) and the y coordinate is zero, this would suggest that the source operates mostly as a vertical load at this particular instance.

SHORT WAVELENGTH RADIATION OF DRILL BITS

Vertical loads

In the previous sections, we treated long wavelengths and substituted a horizontally stratified Earth by a TI medium. We will now consider another extreme case when the wavelengths become comparable or smaller than the thickness of the layers. For these examples, we choose the cut-off frequency 1000 Hz, so the wavelengths now are in the same range as the thicknesses of the layers. Figure 3 shows the radiation pattern from the vertical load. Once again, the wavefront is not spherical.

One can observe strong ringing in the z-direction (resonant codas) and dispersion effects, where different frequencies propagate with different velocities. The dispersion effects are best seen on the bottom row of Figure 3.

Rotational load

Finally, let us consider Figure 4 that depicts the displacement field for the rotational load. In this case, the radiation pattern looks rather similar to the one illustrated in Figure 2. The difference is in the amplitude distribution of the SH-wave over the waveforms. The bottom row in Figure 4 shows the x- and y-components of the displacement field and their dispersive nature.

CONCLUSIONS

In this paper, we emphasized the role of the medium parameters and frequency on the computed radiation patterns of vertical and rotational loads over disk of small radius. Similar to a homogeneous Earth, the radiation patterns are decoupled into rotational (SH) and qP-qSV propagation caused by rotational and vertical loads. On the other hand, there is a big difference in computed radiation patterns compared to a homogeneous and isotropic Earth propagation. In the case of fine horizontal stratification, the medium behaves as a TI (anisotropic) material showing coupling between quasi-compressional and quasi-shear waves. Opposed to this case, when the layers are "thick," radiation is characterized by strong dispersion. We conclude that

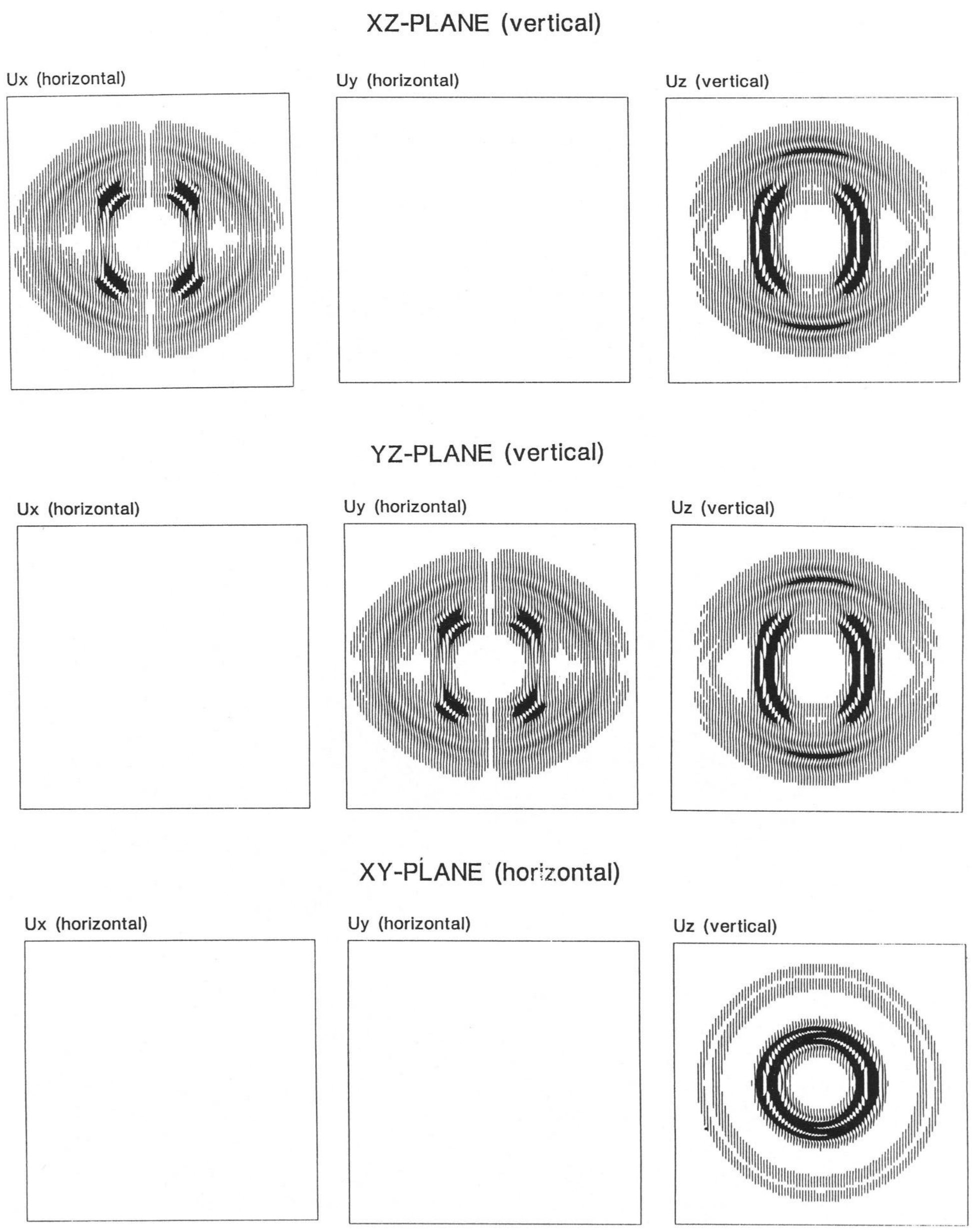

FIG. 1. Three components of the displacement in the XZ, YZ, and XY planes at a fixed propagation time. Vertical load applied at the center of each plot at time $t = 0$. Long wavelength assumption is used.

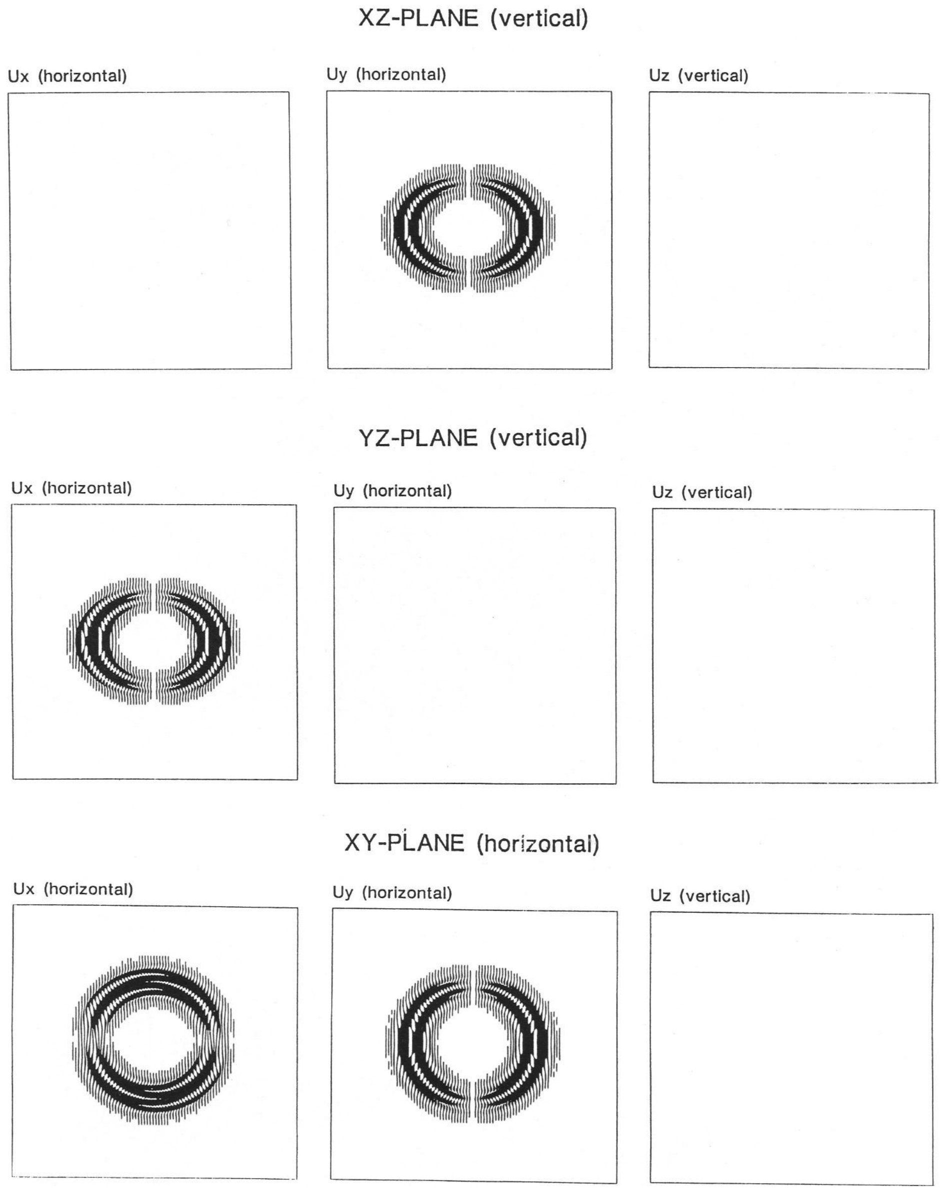

FIG. 2. Three components of the displacement in the XZ, YZ, and XY planes at a fixed propagation time. Rotational load applied at the center of each plot at time $t = 0$. Long wavelength assumption is used.

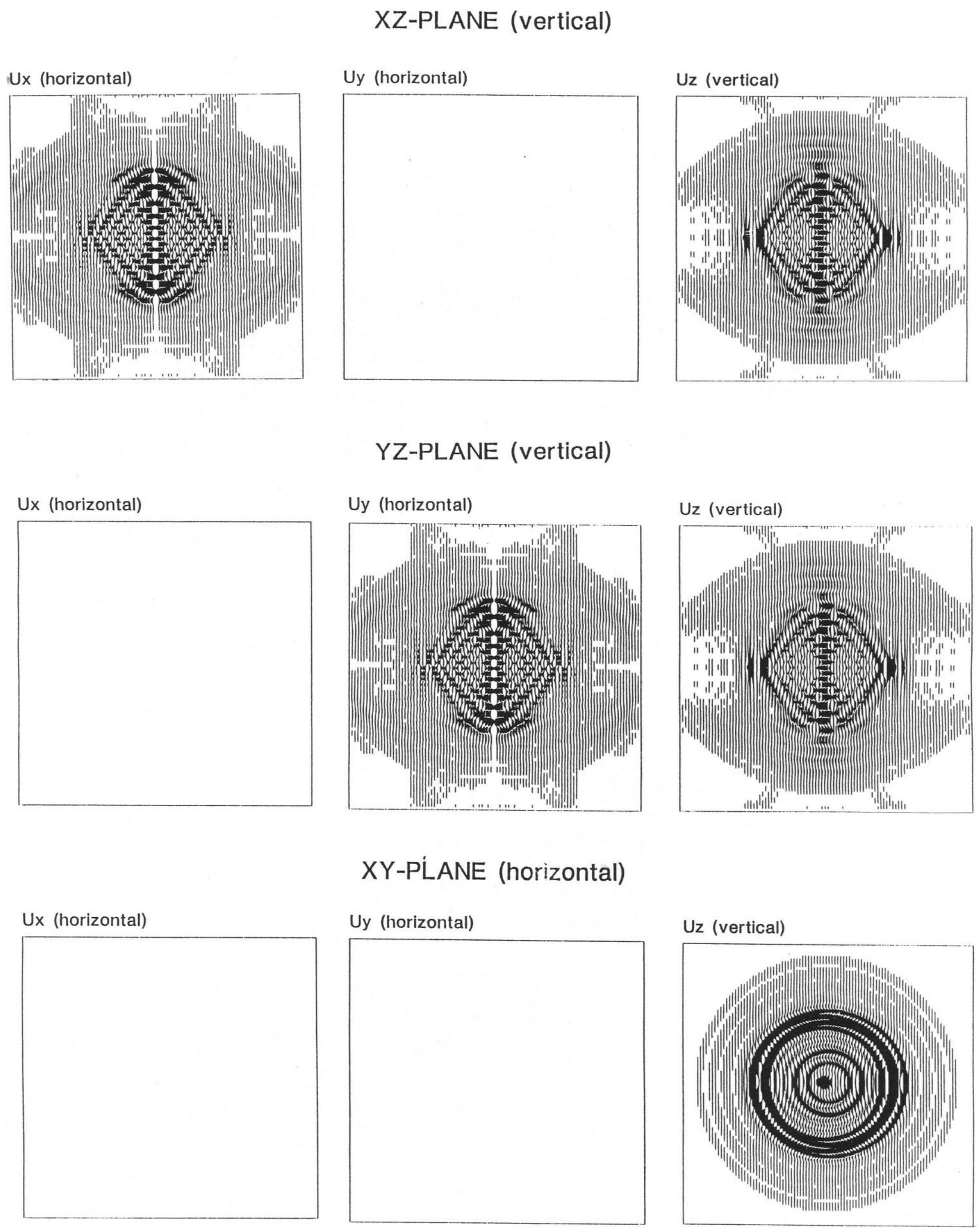

FIG. 3. Three components of the displacement in the XZ, YZ, and XY planes at a fixed propagation time. Vertical load applied at the center of each plot at time $t = 0$. Short wavelength assumption is used.

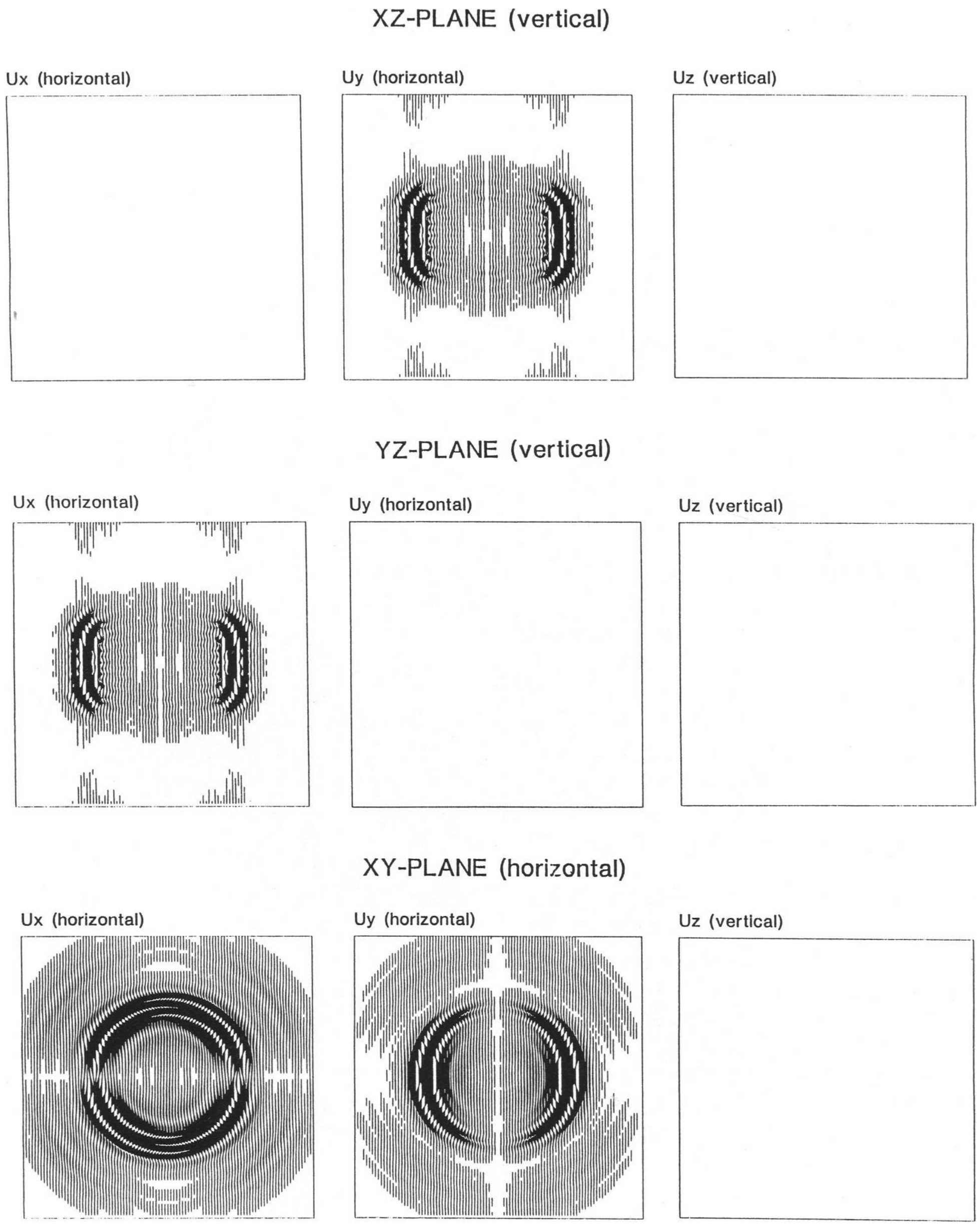

FIG. 4. Three components of the displacement in the XZ, YZ, and XY planes. Rotational load applied at the center of each plot at time $t = 0$. Short wavelength assumption is used.

computed radiation patterns can be important for accurate monitoring of complex seismic sources if a three-component displacement vector is measured at the surface of the Earth.

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