APPLICATION OF FIBONACCI TECHNIQUE TO SEISMIC SECTION MODELING

J. M. CARCIONE AND R. D. ELIA

Departamento Estudios Especiales, Yacimientos Petroliferos Fiscales. Avda. Pte. R. S. Peña 777, Of. 1305. 1364 Buenos Aires, Argentina.

ABSTRACT

A method for modeling seismic sections is presented, whereby primary reflections can be obtained by ray tracing based on the idea of the source horizon. Diffraction effects are obtained with an existing theory based on Kirchhoff's retarded potential method which is generalized in order to be applied to geological layered models. Application of Fibonacci search method to find the rays results in significant computer time reduction.

INTRODUCTION

Since advent of the one-dimensional seismic modeling in the '50s up to date, major changes have occurred concernig both mathematical and computing development and requirements of geophysical interpreters who use seismic modeling as a tool for obtaining subsurface data.

The response of very realistic geological models can be obtained by solving the wave equation with numerical methods but, in general, too much computer time is involved in implementing programs and it does not differ significanty from the ray-theory program.

In this paper we describe a method for seismic section modeling satisfying geophysical interpreters' requirements including an algorithm aimed at speeding up computation. The technique in question, which was already used in Seismics by Khattri et al (1980) on real data for velocity analysis, is Fibonacci search method herein applied for fast raypath finding.

The technique permits us to obtain the seismic response of a layered medium with interfaces of varied geometry, which are approximated by segments with a third degree polynomial. Reflection impulse response is obtained by using the ray theory under the assumption that each subsurface reflector point behaves as a source.

Included are diffraction effects by applying Kirchhoff's scalar wave theory for approximating compressional waves in an elastic medium, which was introduced by Trorey (1970) and Hilterman (1970) into seismic modeling.

DEFINITION OF THE GEOLOGICAL MODEL

The geological model used is a two-dimensional layered medium composed of N homogeneous and isotropic layers. Interfaces are defined by a set of points (x_j, y_j) ; j=0,...n which are adjusted by n-1 polynomials of the form

$$P_j(x) = a_j(x-x_j)^3 + b_j(x-x_j)^2 + c_j(x-x_j) + d_j \quad (1)$$

which applies to the $[x_j, x_{j+1}]$ interval, where coefficients a_j, b_j, c_j and d_j are such that

$$\begin{aligned} &P_j(x_j) = y_j && j = 0,\ldots n \\ &P_{j+1}(x_{j+1}) = P_j(x_{j+1}) && j = 0,\ldots n-2 \\ &P'_{j+1}(x_{j+1}) = P'_j(x_{j+1}) && j = 0,\ldots n-2 \\ &P''_{j+1}(x_{j+1}) = P''_j(x_{j+1}) && j = 0,\ldots n-2 \\ &P''(x_o) = P''(x_n) = 0 && \end{aligned} \quad (2)$$

This method is known as cubic spline interpolation with free boundary conditions (Burden and Faires, 1978).

Each layer is characterized by its velocity of propagation V and its density D, which may vary linearly both horizontally, e.g. by simulating facies changes, and vertically by considering compaction effects. The corresponding expressions are

$$V(x,y) = A\,x + B\,y + C \quad (3)$$

$$D(x,y) = E\,x + F\,y + G \quad (4)$$

THE IMPULSIVE RESPONSE

The method used for describing seismic wave propagation is the ray theory which states that disturbance energy travels along clearly defined raypaths which are always normal to wavefronts. Simplifying the acoustic wave equation is possible when the disturbance wavelength is small as compared with the extent of the change, either in acoustic impedance or in the interface geometry. Obviously, making use of this approximation no wavelength-dependent phenomena, such as diffraction, can be described; these should be approached through the wave equation.

The objetive is to obtain a nonmigrated seismic section, where inelastic attenuation effects, spherical divergence, transmission losses, and other phenomena affecting the disturbance amplitude have been compensated. Thus, it is possible to compare this synthetic seismic section with a seismic section resulting from applying conventional processing methods. Under these conditions, the relative amplitude of the signal received is supposed to depend uniquely on the reflection coefficient magnitude at normal incidence

$$R_k = (Z_{k+1} - Z_k) / (Z_{k+1} + Z_k) \qquad k = 1,\ldots N \tag{5}$$

where

$$Z = D\ V$$

is the acoustic impedance.

The first step is to find the impulsive response to the geological model for a given configuration of surface shot-receiver locations. Each reflection event is identified by the two-way travel time to the reflector and the reflection coefficient magnitude.

Each reflector point is supposed to behave as a source emitting a surfaceward ray which is normal to the interface at such a point. Therefore, two-way travel time is twice the time this ray takes to reach the surface.

RAYPATH COMPUTATION

Raypath computation can be made simpler by adopting a different reference system at each layer, such that the velocity law given in (3) depends only upon one variable, x'. This can be achieved by rotating the original system an angle α, its value being

$$\alpha = \arctan(B/A) \tag{6}$$

Velocity law as referred to the new system is

$$V(x') = R\ x' + C \qquad \text{with} \quad R = \sqrt{A^2 + B^2} \tag{7}$$

The raypath equation for this velocity law is a circumference expressed by

$$(y'-y'_c)^2 + (x'-x'_c)^2 = 1/\ p^2\ R^2 \tag{8}$$

where

$$y'_c = y_i + \cos\theta'_i\ /\ p\ R \tag{9}$$

$$x' = -\ C\ /\ R \tag{10}$$

with

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{11}$$

(x_i, y_i) being the ray starting point coordinates at interface i, as indicated in fig. 1, and

$$\theta'_i = \theta_i - \alpha \tag{12}$$

where θ_i is the angle between the tangent to the ray and the x-axis, finally, p is Snell's parameter as expresed by

$$p = \sin\ \theta'_i\ /\ V_i \tag{13}$$

V_i being the velocity of propagation at point (x_i, y_i).

In the original system, the raypath equation is

$$y = y_c \pm \sqrt{1/\ p^2\ R^2 - (x - x_c)^2} \tag{14}$$

and it is centered at

$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x'_c \\ y'_c \end{bmatrix} \tag{15}$$

the angle between the tangent to the ray and the x-axis at any raypath point being

$$\theta = \alpha + \arcsin\left\{p\left[\ C + R\ (x\cos\alpha + y\sin\alpha)\right]\right\} \tag{16}$$

the travel time expression through the layer is

$$T = \ln \left[(1 + \cos \theta_i') \; (C + R x') \; / \; V_i \; (1 + \sqrt{1 - p^2 \; (C + R x')}) \right] / R \tag{17}$$

In the case in which velocity is constant throughout the layer, i.e. A = B = 0, the right-hand side in equation (8) representing the curvature radius tends to infinite thus verifying that the raypath is a straight line.

DIFFRACTION AMPLITUDE

It is well-known the theory relating subsurface reflector geometry with diffraction amplitude, which was almost simultaneously published by Trorey and Hilterman. This theory makes use of Kirchhoff's retarded potential solution of the acoustic wave equation to estimate a diffraction plate response.

One constraint of this theory would be its application to seismic sections, since it has been developed for zero offset but Berryhill (1977) showed it can be a sufficiently satisfactory approximation provided source-receiver separation of the data to be stacked is not too large.

Based on Hilterman's work, Berryhill developed an expression for obtaining diffraction amplitude for a constant velocity layer, whose notation has been slightly changed in what follows

$$D_o = \arctan \left(\sqrt{1 + 4 \, T_0 \, F_d} \; / \; 2 \, T_0 \, F_d \, \sin \phi_o \right) \times \cos \phi_o \; / \; \pi \tag{18}$$

where F_d is the seismic signal dominant frequency, T_o is the diffractor point-receiver point travel time, and ϕ_o is the angle measured between the normal to the reflecting plane and the least timepath to such a plane edge.

In order to apply expression (18) to the more general case of a layered medium as previously defined, the image reflector concept is used.

Image reflector $\overline{A'D'}$, as shown in fig. 2, is the reflector segment which, when seen from receiver point G, has the same effects as the real horizon segment $\overline{AD}$, the layered medium being substituted by a layer of unique velocity, the one corresponding to the G point. That is, the ray arrival slope from D' and the travel time from D' to G are the same as those obtained for the diffractor point D. The same applies to point A' an image of the auxiliary point A.

Under these considerations, the angle ϕ_o in expression (18) is evaluated by

$$\phi_o = (T_o - T_o' \cos \beta) \;/\; T_o' \sin \beta \qquad (19)$$

where β is the angle between the rays emerging from D' and A' and T_o' is the travel time from A to the receiver point.

APPLICATION OF FIBONACCI TECHNIQUE TO OBTAIN THE OPTIMUM RAY

Fibonacci search method is an algorithm for minimizing a strictly quasiconvex funtion over a closed interval, based on Fibonacci sequence, as defined by

$$F_{k+1} = F_k + F_{k-1} \qquad k = 1, 2, \ldots\ldots \qquad (20)$$

with

$$F_o = F_1 = 1$$

The objective is to find the ray which, starting from horizon i, arrives at receiver point G with an error less than a predeterminated ε.

The initial interval of uncertainty for application of the method is defined by two points x_{a1} and x_{b1} generating arrivals X_{a1} and X_{b1} which are located on opposite sides as related to point G (see fig. 3).

The number of iterations, q-2, is such that

$$F_q > (x_{a1} - x_{b1}) \;/\; \delta \qquad (21)$$

where δ represents the final interval of uncertainty.

Consider two points on the reflector horizon i given by

$$l_k = x_{ak} + (x_{bk} - x_{ak})\ F_{q-k-1} / F_{q-k+1} \qquad (22)$$

$$m_k = x_{bk} + (x_{bk} - x_{ak})\ F_{q-k}\ / F_{q-k+1} \qquad (23)$$

for $k = 1,\ldots q-1$

The new interval of uncertainty $[x_{a(k+1)}, x_{b(k+1)}]$ is given by $[l_k, x_{bk}]$ if $L_k > M_k$ and by $[x_{ak}, m_k]$ if $L_k \leqq M_k$ (Bazaraa and Shetty, 1979). Only at the first iteration should raypaths for m_1 and l_1 be calculated; at each of the subsequent iterations it is only necessary to make one evaluation. Thus, q-1 raypath computations are required for the q-2 iterations.

The procedure continues until any of the functions to be minimized satisfies

$$\psi_a = |X_{ak} - X_G| < \varepsilon \qquad (24)$$

$$\psi_b = |X_{bk} - X\ | < \varepsilon \qquad (25)$$

or until making iteration q-2, where the mid-point fo the interval of uncertainty is taken as the value searched for.

It is applied in a similar way to computation of diffraction times, taking the ray starting angle with respect to the x-axis as the variable.

CONCLUSIONS

A FORTRAN IV program has been implemented on a Hewlett Pac kard 3000 computer using Fibonacci technique. Getting the impulse response for 100 traces of the geological model shown in fig.4, which consists of 7 interfaces and 4 diffractor points, took less than 10 minutes of computation using this program. The error involved in rays reaching receivers is on the order of 1% the receiver distance.

Application of the program to different geological models has shown that the method used resulted in significant computer time reduction, as compared with conventional seismic section modeling programs.

As pointed out above, the algoritm is convergent if the function to be minimized is strictly quasiconvex. As can be easily seen, this does not strictly hold for most of the subsurface models, except for simple cases; the convexity of the function to be minimized is shown in the Appendix for a geological model consisting of plane and infinite interfaces. Fortunately, the geometry of the interfaces used for the realistic models is such that it allows application of the technique in most cases. When this is not possible, source points spacing on the reflector is reduced in order to obtain a surface degree of precision similar to the Fibonacci method.

An important decision is choosing the initial interval of uncertainty $[x_{a1}, x_{b1}]$, which should be small enough for the function to satisfy convexity conditions and large enough for the technique to be considerably time-saving.

APPENDIX

Consider a medium as shown in fig.5, with constant velocity layer and plane, infinite interfaces given by

$$y = a_j x + b_j \qquad j = 0, \dots N$$

The coordinate of the receiver point X in terms of that of source point x at any interface i, is

$$X = H_i x + Q_i + q_1 \tag{A-1}$$

where

$$H_i = \prod_{j=1}^{i} h_j \qquad Q_i = \sum_{k=2}^{i} q_k \prod_{j=1}^{k-1} h_j$$

with

$$h_j = (p_j + a_j) / (p_j - a_{j-1})$$

$$q_j = (b_j + b_{j-1}) / (p_j - a_{j-1})$$

Slope p_j of the ray transmitted at the j interface is obtained by applying Snell's laws

$$p_j = \tan\left\{ \arcsin\left[\sin(\gamma_j - \arctan p_{j+1})\ V_j \right.\right.$$
$$\left.\left. /\ V_{j-1} \right] - \gamma_j \right\}$$

with

$$\gamma_j = \arctan(1\ /\ a_j) \qquad\qquad j = 1, \ldots m-1$$

The starting slope is

$$p_i = -1\ /\ a_i$$

Using (A-1), the function given in (24) and (25) is expressed as

$$\psi(x_i) = \left| A_i x_i + B_i \right| \tag{A-2}$$

with

$$A_i = H_i \qquad\qquad B_i = \Omega_i + q_1 - X_G$$

According to definition, the function given in (A-2) is convex if

$$\psi\left[\lambda\ x_a + (1 - \lambda)\ x_b \right] \leqslant \lambda\psi(x_a) + (1 - \lambda)\ \psi(x_b) \tag{A-3}$$

for each pair of source points x_a, x_b and each λ (0,1).

We have

$$\psi\left[\lambda\ x_a + (1 - \lambda)\ x_b \right] = \left| A_i x_a \lambda + A_i\ (1 - \lambda)\ x_b + B_i \right|$$
$$\leqslant \lambda \left| A_i\ x_a + B_i \right| + (1 - \lambda) \left| A_i x_b + B_i \right|$$

which is equivalent to (A-3), showing that ψ is a convex function. This means that the function is strictly quasiconvex, which is a necessary condition for application of Fibonacci method.

REFERENCES

BAZARAA, M. S., and SHETTY, C.M., 1979, Non linear programming, John Wiley & Sons, Inc., New York.

BERRYHILL, J.R., 1977, Diffraction response for nonzero separation of source and receiver, Geophysics 42, 1158-1176.

BURDEN, R. L., FAIRES, J. D. and REYNOLDS, A. C., 1978, Numerical analysis, Prindle, Weber & Schmidt, Boston.

HILTERMAN, F. J., 1970, Three-dimensional seismic modeling, Geophysics 35, 1020-1037.

KHATTRI, K., VIG, P. K. and RAO, N. D. J. 1980, An optimized technique for determining stacking velocity from seismic reflection data, Geophysical Prospecting 28, 825-839.

TROREY, A. W., 1970, A simple theory for seismic diffractions, Geophysics 35, 762-784.

LIST OF FIGURE CAPTIONS

1- Changing the refererence system allows making raypath computation simpler.

2- Diffraction computation. A multivelocity medium can be represented by a single layer through the reflector image.

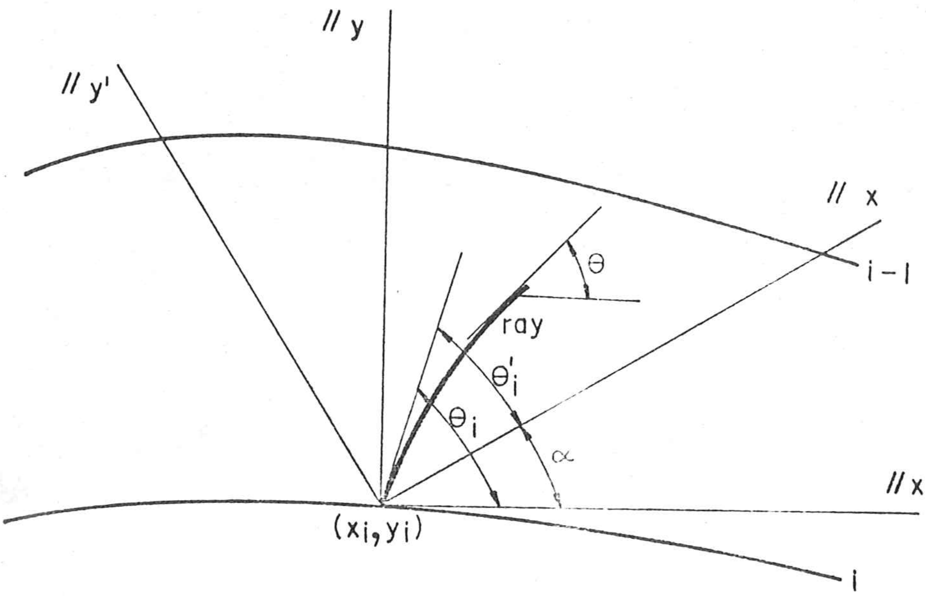
3- Subsurface model with ray tracing for application of Fibonacci technique.

4- Constant density geological model used to check the velocity of the method proposed.

5- Simple geological model for which the function to be minimized is convex.

CARCIONE-ELIA

FIG. 1



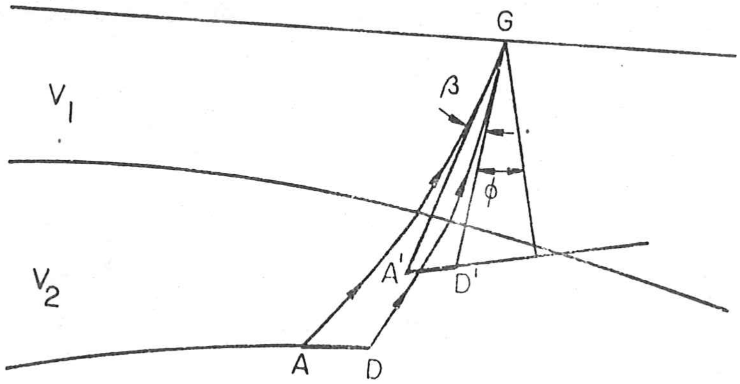
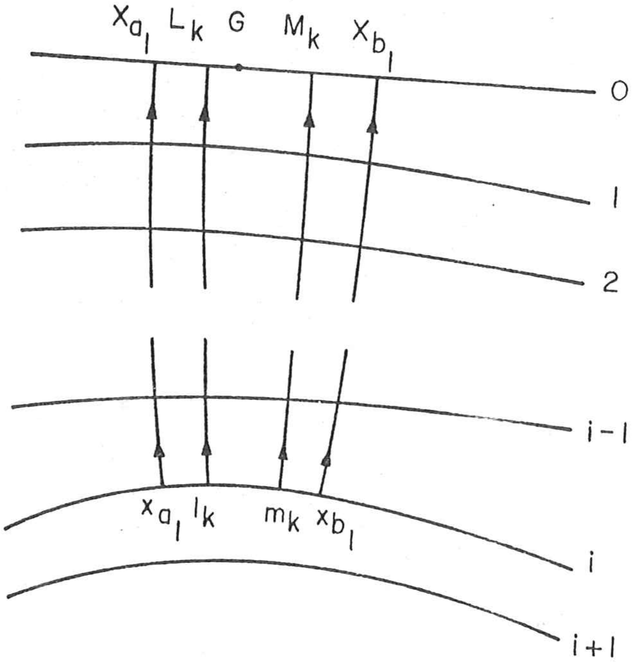


FIG 2 CARCIONE-ELIA

CARCIONE-ELIA

FIG. 3



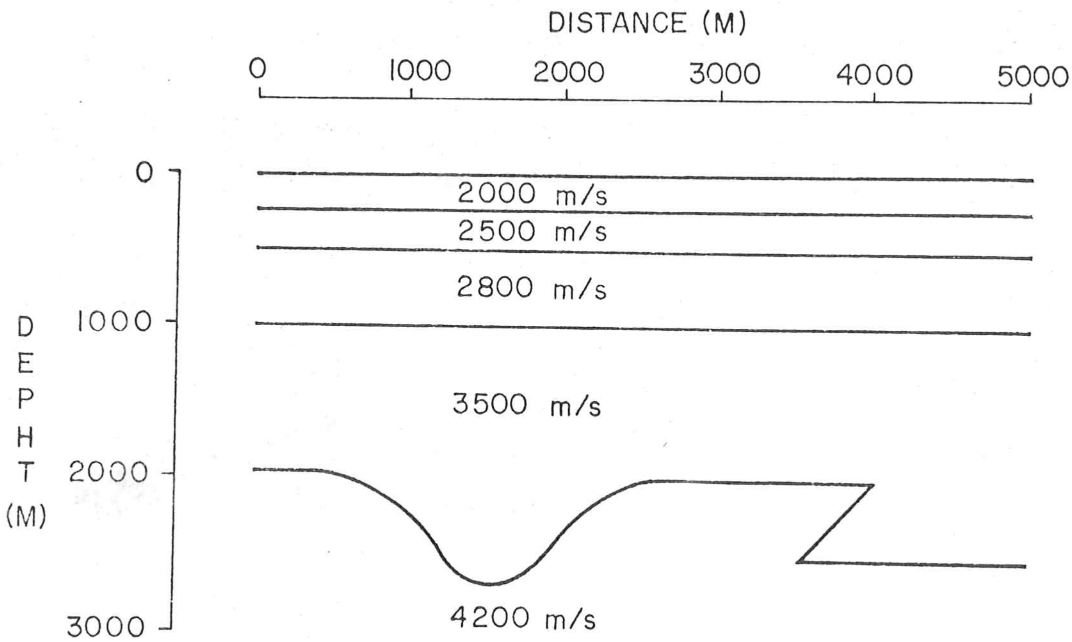


FIG. 4 CARCIONE-ELIA

FIG. 5

CARCIONE-ELIA