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A constitutive equation and generalized Gassmann modulus for multimineral porous media

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ABSTRACT

We derive the time-domain stress-strain relation for a porous medium composed of $n - 1$ solid frames and a saturating fluid. The relation holds for nonuniform porosity and can be used for numerical simulation of wave propagation. The strain-energy density can be expressed in such a way that the two phases (solid and fluid) can be mathematically equivalent. From this simplified expression of strain energy, we analogize two-, three-, and n-phase porous media and obtain the corresponding coefficients (stiffnesses).

Moreover, we obtain an approximation for the generalized Gassmann modulus. The Gassmann modulus is the bulk modulus of a saturated porous medium whose matrix (frame) is homogeneous. That is, the medium consists of two homogeneous constituents: a mineral composing the frame and a fluid. Gassmann's modulus is obtained at the low-frequency limit of Biot's theory of poroelasticity. Here, we assume that all constituents move in phase, a condition similar to the dynamic compatibility condition used by Biot, by which the P-wave velocity is equal to Gassmann's velocity at all frequencies.

Our results are compared with those of the Berryman-Milton (BM) model, which provides an exact generalization of Gassmann's modulus to the three-phase case. The model is then compared to the wet-rock moduli obtained by static finite-element simulations on digitized images of microstructure and is used to fit experimental data for shaly sandstones. Finally, an example of a multimineral rock ($n > 3$) saturated with different fluids is given.

INTRODUCTION

We obtain the stress-strain relation and propose a generalization of Gassmann's bulk modulus (Gassmann, 1951) for a multimineral porous medium. Rocks such as sandstones are rarely clean because they may contain clay, feldspar, and dolomite. In the case of gas-hydrate–bearing sediments, the rock (or sediment) can be composed of three solids: quartz, clay, and gas hydrate (Carcione and Seriani, 2001).

The derivation of the effective modulus uses a generalization of the strain-energy density from the two- and three-phase cases to n phases. Biot (1956) has obtained Gassmann's velocity at low frequencies from the dispersion relation (the reference velocity V_c using Biot's notation; see his equations 5.4, 7.21, and 7.22). Biot (1956) has obtained a dynamic compatibility condition, relating the stiffness moduli and the mass-density coefficients, by which the P-wave velocity is equal to Gassmann's velocity at all frequencies. This condition is equivalent to impose wave propagation without relative motion between fluid and solid [see also Biot (1962), his equation 8.29]. Biot's theory has been generalized to three phases — two solid and one fluid — by Leclaire (1992) and Leclaire et al. (1994, 1995), who confirm it with laboratory experiments. As before, the solution of the P-wave dispersion relation at low frequencies can be obtained alternatively from the stress-strain relation by assuming the three phases move as a whole. In this work, we generalize the theory to n phases ($n - 1$ solids and a fluid) and obtain the low-frequency modulus, analogous to the two- and three-phase theories, by assuming no relative motion between the single constituents of the medium.

The n-phase porous medium implies a particular topological configuration — namely, one where the solid phases form $n - 1$ continuous and interpenetrating solid matrices. A typical example is permafrost, where ice forms with decreasing

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temperature and generates a frame within the main skeleton of the rock. This configuration is different from that assumed by Gurevich and Carcione (2000) and Berryman and Milton (1991), where the two frames are not overlapping. As in Biot's (1956) and Leclaire's (1992) theories, there is no assumption on the geometry of the grains and pore space, which can have any shape. Carcione et al. (2000) apply this theory to study the acoustic properties of shaley sandstones, assuming that sand and clay are not welded. Both frozen media and shaley sandstones are examples of porous materials where the two solid phases are weakly coupled or unwelded. The unwelded condition between the two solid phases is assumed when the potential and kinetic energies are defined, with proper interaction terms between the solid and fluid phases. If the two solid phases were welded, then we would not get additional slow waves, as in Brown and Korringa (1975). As in Biot's theory, the present model cannot be directly used to describe the anelastic effects (attenuation and velocity dispersion) related to patchy composites, such as nonuniform gas saturation (White, 1975). However, these effects can be described by using numerical modeling techniques based on the differential equations of our theory (e.g., Carcione et al., 2003a, b).

Our analysis provides the time-domain stress-strain relation for nonuniform porosity, which can be used to compute synthetic seismograms by direct methods. A similar extension of Gassmann's modulus to n fluids is not required because the effective modulus of the fluid mixture can be obtained with the Wood model (e.g., Mavko et al., 1998). Although capillarity effects may be responsible for additional slow waves (Santos et al., 1990), they have little influence on the magnitude of the bulk modulus of a homogeneous medium (Carcione et al., 2004). However, the situation is different if patchy saturated (inhomogeneous) porous media are considered. In this case, capillarity effects can be important (Tserkovnyak and Johnson, 2003).

We compare our model with that of Berryman and Milton (1991), who generalize Gassmann's equation to three-phase porous media. They use the equation obtained by Brown and Korringa (1975) for conglomerates and derive exact expressions for the two elastic moduli associated with changes in the bulk and pore volumes of the medium. These moduli are properties of the composite solid frame and depend on the bulk moduli, dry-rock moduli, and porosity of each phase. A more realistic test is a comparison with finite-element simulations of the bulk modulus (Arns, 2002). Arns uses the finite-element method to compute the Gassmann modulus from static numerical experiments. He explicitly models porous media with different morphologies simulating the structure of real rocks. Arns reports that Krief's empirical model (Carcione et al., 2000; Gurevich and Carcione, 2000) successfully describes the dry-rock moduli of consolidated clastic rocks.

The first section of this paper establishes the analogy between the two-phase medium and the n-phase medium to obtain the elastic moduli of the strain-energy density. The generalized Gassmann modulus and the stress-strain relation are obtained in the next section. Then, a model for the dry-rock moduli is introduced. Finally, our theory is compared to the three-phase Berryman-Milton (BM) model, Arns' simulations, and experimental data.

STRAIN ENERGY

The main assumptions of our theory are (1) the deformations are infinitesimal, (2) the principles of continuum mechanics can be applied to measurable macroscopic variables, (3) the conditions are isothermal, (4) the stress distribution in the fluid is hydrostatic, (5) the liquid phase is continuous, (6) the materials of the frames are isotropic, and (7) the medium is statistically isotropic and fully saturated (Biot, 1956; Leclaire, 1994).

Let us consider an elementary volume of porous material composed of $n-1$ solid phases and a fluid phase denoted by subscript n. If Ω_i are the partial volumes and Ω is the total volume, the fraction of solid i is $\phi_i = \Omega_i/\Omega$ and

$$\sum_{i=1}^{n-1} \phi_i + \phi_n = 1, \tag{1}$$

where ϕ_n is the porosity. We derive the stress-strain relation from the strain-energy density. We consider only dilatations, since the shear modulus is not affected by the presence of the fluid [dry- and wet-rock shear moduli are equal (Berryman, 1999)]. The general case, including the shear terms, is given in the next section.

In the linear isotropic case, the strain-energy density is a quadratic positive definite form in the strain invariants. For pure dilatational deformations, it can be defined as

$$V = \frac{1}{2}\sum_{i=1}^{n} B_i \theta_i^2 + \sum_{i=1}^{n}\sum_{j>i}^{n} C_{ij}\theta_i\theta_j, \tag{2}$$

where θ_i are the invariants (dilatations; see Appendix A) and B_i and C_{ij} are stiffness (elastic) moduli. This form of strain energy considers the energy of the single constituents (first term) and the interaction energies (second term) and is consistent with Biot's (1956) theory of poroelasticity.

To evaluate the moduli B_i and C_{ij}, we generalize the elastic moduli of Biot's theory ($n = 2$) (Mavko et al., 1998; Carcione, 2001). Denoting the solid with index 1 and the fluid with index 2, Biot's moduli are

$$\begin{aligned} B_1 &= K_{m1} + (\alpha_1 + \phi_1 - \beta_1)^2 M, \\ C_{12} &= (\alpha_1 + \phi_1 - \beta_1)\phi_2 M, \\ B_2 &= \phi_2^2 M, \end{aligned} \tag{3}$$

where

$$M = \left(\frac{\alpha_1 + \phi_1 - \beta_1}{K_1} + \frac{\phi_2}{K_2}\right)^{-1}, \tag{4}$$

$$\alpha_1 = \beta_1 - \frac{K_{m1}}{K_1}, \tag{5}$$

and

$$\beta_1 = \frac{\phi_1}{1-\phi_2} = 1. \tag{6}$$

Here, K_1 and K_2 are the solid and fluid bulk moduli and K_{m1} is the dry-rock bulk modulus, where the subscript m denotes matrix. The reason for using the trivial expression $\beta_1 = 1$ and the form of α_1 is clarified below, where we generalize these equations to n solid phases. The expression of the Gassmann modulus is

$$K_G = K_{m1} + \alpha_1^2 M \tag{7}$$

(Mavko et al., 1998; Carcione, 2001). If $K_{m1} = 0$, that is, if the medium is a suspension of solid particles in the fluid, by combining equations 4, 5, and 7 we obtain the Wood modulus

$$K_G = \left(\frac{1-\phi_2}{K_1} + \frac{\phi_2}{K_2}\right)^{-1} \tag{8}$$

(Mavko et al., 1998).

Note that equations 3–5 correspond to an effective solid porosity

$$\phi_1' = \alpha_1 + \phi_1 - \beta_1, \tag{9}$$

such that

$$\begin{aligned} B_1 &= K_{m1} + \phi_1'^2 M, \\ C_{12} &= \phi_1' \phi_2 M, \\ B_2 &= \phi_2^2 M, \end{aligned} \tag{10}$$

and

$$M = \left(\frac{\phi_1'}{K_1} + \frac{\phi_2}{K_2}\right)^{-1}. \tag{11}$$

At this point, in view of the simple form of equations 10, the generalization to $n-1$ solid phases is straightforward by analogy. The corresponding equations are

$$\begin{aligned} B_i &= K_{mi} + \phi_i'^2 M, \quad i = 1, \ldots, n-1, \\ C_{ij} &= \phi_i' \phi_j' M, \quad i < j, \\ B_n &= \phi_n^2 M, \end{aligned} \tag{12}$$

where $\phi_n' = \phi_n$ in the second expression,

$$M = \left(\sum_{i=1}^{n-1} \frac{\phi_i'}{K_i} + \frac{\phi_n}{K_n}\right)^{-1}, \tag{13}$$

and

$$\phi_i' = \alpha_i + \phi_i - \beta_i = \alpha_i - \beta_i \phi_n. \tag{14}$$

In equation 14,

$$\alpha_i = \beta_i - \frac{K_{mi}}{K_i}, \tag{15}$$

and the value

$$\beta_i = \frac{\phi_i}{1-\phi_n} \tag{16}$$

is the fraction of solid i per unit volume of total solid. Here, K_i, $i = 1, \ldots, n-1$ and K_n are the solid and fluid bulk moduli, respectively, and K_{mi}, $i = 1, \ldots, n-1$ are the frame moduli. As we shall see later, these moduli depend on β_i. Therefore, they are not an intrinsic-intensive property of phase i as in the BM model, which considers separate porous matrices. This analogy between the two-phase porous medium and the n-phase porous medium follows from Leclaire (1992), who obtains in this way the stiffness moduli of a three-phase frozen porous medium.

As mentioned above, we need to justify expressions 5 and 15 for α_i. These expressions are obtained in Appendix A, where the strain energy is written in terms of the variation of fluid content ζ (see equation A-2). The physical reason behind this justification is that the formulation based on ζ is compatible with experiments and allows us to establish the stress-strain relation for nonuniform porosity (Biot, 1962; Carcione, 2001).

STRESS-STRAIN RELATION AND GENERALIZED GASSMANN'S MODULUS

First, let us clarify the meaning of the different averaged stress components that we use in the demonstration. Assume that the indices k and l denote the Cartesian components. ($i = 1, \ldots, n-1$ refer to the solid phases.) Define $s_{kl}^{(i)}$ as the average value of the stress tensor over Ω_i. Then, the contribution of the ith solid to the total stress tensor is $\sigma_{kl}^{(i)} = \phi_i s_{kl}^{(i)}$. We introduce the averaged components per unit volume of solids $\tau_{kl}^{(i)}$ through the relation $\sigma_{kl}^{(i)} = \beta_i \tau_{kl}^{(i)}$. Hence, $\sigma_{kl}^{(i)}$ are partial stress components of phase i and $s_{kl}^{(i)}$ are the intrinsic stress components. In Appendix A, $\tau_{kl}^{(i)}$ are used to obtain the stress-strain relations for nonuniform porosity.

Let us define the mean stress corresponding to each solid phase as $\sigma_i = \sigma_{kk}^{(i)}/3$, where the Einstein summation convention of repeated indices is used. Having defined the strain-energy expression in equation 2, we calculate, the mean stress of each phase as

$$\sigma_i = \frac{\partial V}{\partial \theta_i}. \tag{17}$$

Defining $C_{ij} = C_{ji}$, we obtain

$$\sigma_i = B_i \theta_i + \sum_{j=1 (j \neq i)}^{n} C_{ij} \theta_j. \tag{18}$$

The mean total stress is given by

$$\sigma = \sum_{i=1}^{n} \sigma_i = \sum_{i=1}^{n} B_i \theta_i + \sum_{i=1}^{n} \sum_{j=1 (j \neq i)}^{n} C_{ij} \theta_j. \tag{19}$$

As Biot (1956) ($n = 2$) and Leclaire (1992) ($n = 3$) did, we now prevent any relative motion between the different constituents of the porous medium to obtain the low-frequency modulus. Then, $\theta_i \equiv \theta,\ i = 1, \ldots, n$ and stress-strain relation 19 becomes

$$\sigma = \left(\sum_{i=1}^{n} B_i + \sum_{i=1}^{n} \sum_{j=1 (j \neq i)}^{n} C_{ij}\right) \theta, \tag{20}$$

where we identify the Gassmann modulus as

$$K_G = \sum_{i=1}^{n} B_i + \sum_{i=1}^{n} \sum_{j=1 (j \neq i)}^{n} C_{ij}. \tag{21}$$

Substituting equations 12 into equation 21 yields

$$K_G = \sum_{i=1}^{n-1} K_{mi} + M \sum_{i=1}^{n} \phi_i'^2 + M \sum_{i=1}^{n} \sum_{j=1 (j \neq i)}^{n} \phi_i' \phi_j'. \tag{22}$$

Equation 22 simplifies to

$$K_G = \sum_{i=1}^{n-1} K_{mi} + M \left[\sum_{i=1}^{n-1} (\alpha_i + \phi_i - \beta_i) \right]^2, \tag{23}$$

where we use equation 14. Using equation 1 and because $\sum_{i=1}^{n-1} \beta_i = 1$, we finally get the expression for the generalized Gassmann modulus:

$$K_G = K_m + \left(\sum_{i=1}^{n-1} \alpha_i \right)^2 M, \tag{24}$$

where M is given by equation 13, and where

$$K_m = \sum_{i=1}^{n-1} K_{mi} \tag{25}$$

is the composite dry-rock modulus, as we show in the next section.

Alternatively, equation 24 can be obtained by imposing motion of the system as a whole in equation A-15. Note that ζ, defined in equation A-2, equals zero in this case (the condition for a closed system) since $\sum_{i=1}^{n-1} \beta_i = 1$. We obtain

$$K_G = \sum_{i=1}^{n-1} K_{mi} + M \sum_{i=1}^{n-1} \alpha_i^2 + 2M \sum_{i=1}^{n-1} \sum_{j>i}^{n-1} \alpha_i \alpha_j, \tag{26}$$

where we use equation A-13. Equation 26 can easily be expressed as equation 24.

The model is appropriate for an unconsolidated material (e.g., loose mixtures of sands and clay). If all of the minerals are suspended in the fluid, $K_{mi} = 0, \alpha_i = \beta_i, \phi_i' = \phi_i$. And because $\sum_{i=1}^{n-1} \beta_i = 1$, we obtain $K_G = M$, i.e., the Wood modulus. If there were random thin dikes of a weak material present in a small volume fraction, the model would predict a decrease in the overall bulk moduli. Assuming these dikes do not form a frame (the corresponding partial dry-rock modulus is zero), the low intrinsic bulk modulus of these intrusions will cause M—and, consequently, K_G—to decrease (see equation 13).

Constitutive equation for nonuniform porosity and implications for wave propagation

The stress-strain relation for nonuniform porosity, generalizing Biot's (1962) constitutive equation to n minerals, is derived in Appendix A. It is given by equations A-12 and A-24:

$$\begin{aligned} \sigma_{kl}^{(i)} = & \left[(K_{Gi} - \phi_n \alpha_i \beta_i M)\theta_i + M(\alpha_i - \phi_n \beta_i) \right. \\ & \left. \times \left(\sum_{m=1(m\neq i)}^{n-1} \alpha_m \theta_m - \zeta \right) \right] + 2\mu_i d_{kl}^{(i)} + \sum_{j\neq i}^{n-1} \mu_{ij} d_{kl}^{(j)}, \\ & \qquad i = 1, \ldots, n-1. \\ p_f = & M \left(\zeta - \sum_{m=1}^{n-1} \alpha_m \theta_m \right), \end{aligned} \tag{27}$$

where p_f is fluid pressure, ζ is variation of fluid content (see equation A-2), $d_{kl}^{(i)}$ are components of the deviatoric strain tensor (see equation A-22), and K_{Gi} are partial Gassmann's moduli given by equation A-13. Use of the constitutive equation 27, together with the Lagrange equations, yields the equation of motion for a multimineral porous medium saturated with a fluid. In such a medium, n P-wave modes and $(n-1)$ S-wave modes propagate; two of these waves are the usual fast waves whose particle motion consists of in-phase movement of all components. The other modes are Biot-type slow waves or static diffusive modes, depending on the frequency range. The additional ith P-wave mode arises from the out-of-phase motion between the ith frame and the fluid, and the slow S-wave modes are caused by the out-of-phase motions of the frames. The importance of this constitutive equation resides in the fact that the presence of the slow wave modes affects the propagation of the fast waves by mode conversion. Therefore, additional relaxation mechanisms causing attenuation and velocity dispersion are activated (e.g., Gurevich and Lopatnikov, 1995; Carcione et al., 2003a). The $n = 3$ case is given in Carcione et al. (2000) and Carcione and Seriani (2001), where three P-waves and two S-waves propagate. The experimental verification of multiwave propagation in a frozen porous medium is given in Leclaire (1995).

MODEL FOR THE DRY-ROCK MODULI

Use of Gassmann's equation 7 requires that one know the dry-rock moduli. There are many models to describe these moduli. Krief et al. (1990) have introduced a model (hereafter called Krief's model) consistent with the concept of critical porosity, since the moduli should vanish above a certain value of the porosity (usually from 0.4 to 0.5). Goldberg and Gurevich (1998), Carcione et al. (2000), and Arns (2002) confirm that Krief's empirical model is successful at describing the dry-rock moduli of consolidated sandstones. The contribution of these moduli depends on the location of the minerals in the porous medium. For instance, calcite can cement quartz grains if deposited during sedimentation under certain conditions of pressure and temperature. On the other hand, calcite particles can be in suspension in the saturating fluid. In the former case, calcite contributes to the stiffness of the rock frame, while in the latter case the contribution is zero. These conditions are taken into account.

A suitable generalization of Krief's model for a multimineral porous medium is given by

$$K_{mi} = \left(\frac{K_{\mathrm{HS}}}{v} \right) \beta_i K_i (1 - \phi_n)^{A/(1-\phi_n)}, \quad i = 1, \ldots, n-1, \tag{28}$$

where A is a dimensionless parameter, $v = \sum_{i=1}^{n-1} \beta_i K_i$ is the Voigt average, and

$$K_{\mathrm{HS}} = \frac{(K_+ + K_-)}{2}. \tag{29}$$

Here, K_+ and K_- are the Hashin-Shtrikman (HS) upper and lower bounds (Hashin and Shtrikman, 1963). For two minerals ($n = 3$), these bounds are given by

$$K_{\pm} = K_1 + \beta_2 \left[(K_2 - K_1)^{-1} + \beta_1 \left(K_1 + 4\frac{\mu_1}{3} \right)^{-1} \right]^{-1} \tag{30}$$

(Mavko et al., 1998), where the upper and lower bounds are obtained by interchanging subscripts 1 and 2 and vice versa. Generally, the expression gives the upper bound when the stiffest material is termed 1 and the lower bound when the softer material is termed 1. For $n > 2$, the general form of these bounds is given by Berryman (1995) and Mavko et al. (1998).

A justification of equation 28 follows. Hashin and Shtrikman (1963) show that for a solid proportion, β_i, K_{mi} is always less than $\beta_i K_i$ (Coussy, 1995). At this upper limit, Gassmann's bulk modulus tends to the Voigt average (Nolen-Hoeksema, 2000). Expression 28 is chosen such that K_m is consistent with the HS bounds when $\phi_n = 0$. The average (equation 29) is a good choice, since the bounds are generally very tight. When $n = 2$, i.e., one mineral, then $K_{HS} = v = K_1$, $\beta_1 = 1$, and the original Krief expression is obtained. This is also the case if all phases are identical. When the porous medium is dry ($K_n = 0$), $M \to 0$ and we have

$$K_G \equiv K_m = \sum_{i=1}^{n-1} K_{mi} = K_{\mathrm{HS}}(1-\phi_n)^{A/(1-\phi_n)}. \quad (31)$$

(The first equality in equation 31 is also obtained in Appendix A by assuming drained conditions.) Finally, it is easy to show from equations 28 and 31 that, as expected, the frame moduli satisfy

$$K_{mi} = 0 \quad \text{for} \quad \phi_i = 0 (\beta_i = 0),$$
$$\sum_{i=1}^{n-1} K_{mi} = K_1 \quad \text{for} \quad \phi_n = 0, \quad \text{and} \quad K_i = K_1,$$
$$i = 1, \ldots, n-1. \quad (32)$$

Actually, in the vicinity of $\phi_n = 0$, the value of K_G is slightly greater than K_{HS}, and there is a discontinuity between K_G and K_m since $K_G(\phi_n = 0) = K_{\mathrm{HS}} + (1 - K_{\mathrm{HS}}/v)w$, where $w = \sum_{i=1}^{n-1} \beta_i / K_i$ is the Wood modulus. The discontinuity at zero porosity does not occur in practice since K_n is never equal to zero (air at room conditions has a bulk modulus of nearly 0.117 MPa).

When the ith mineral does not contribute to the stiffness of the frame because it is in suspension in the fluid, we assume $K_{mi} = 0$.

As an alternative to Krief's model, the critical porosity model can be used (e.g., Mavko et al., 1998). A suitable generalization of this model is

$$K_{mi} = \left(\frac{K_{\mathrm{HS}}}{v}\right) \beta_i K_i \left(1 - \frac{\phi_n}{\phi_c}\right)^{\gamma} \quad i = 1, \ldots, n-1, \quad (33)$$

where ϕ_c is the critical porosity and γ is a coefficient introduced by Roberts and Garboczi (2000), who use it to model the elastic properties of overlapping sphere packs under dry conditions ($\gamma = 1$ in the classical model).

EXAMPLES

There are several models for the case of two solid frames and a saturating fluid. Applications of the three-phase case to wave propagation can be found in (Carcione et al. (2000), Carcione and Seriani (2001), and Carcione et al. (2003b). For a three-phase medium, the Gassmann modulus (equation 24) is

$$K_G = K_{m1} + K_{m2} + (\alpha_1 + \alpha_2)^2 M, \quad (34)$$

where

$$M = \left(\frac{\alpha_1 - \beta_1\phi_3}{K_1} + \frac{\alpha_2 - \beta_2\phi_3}{K_2} + \frac{\phi_3}{K_3}\right)^{-1} \quad (35)$$

(Leclaire et al., 1994). We compare the predictions of our model with those of the three-phase ($n = 3$) BM exact model (Berryman and Milton, 1991; Mavko et al., 1998) and finite-element evaluations of the bulk modulus (Arns, 2002).

Berryman-Milton model

Berryman and Milton's wet-rock (Gassmann) moduli of a three-phase medium is

$$K_G = K_m + \alpha^2 M, \quad (36)$$

$$\frac{1}{M} = \frac{\alpha}{K_s} + \phi_3\left(\frac{1}{K_3} - \frac{1}{K_\phi}\right), \quad (37)$$

$$\alpha = 1 - \frac{K_m}{K_s}, \quad (38)$$

where K_m is the bulk modulus of the dry composite matrix, K_3 is the fluid modulus, and K_s and K_ϕ are constants that depend on the moduli of the single constituents and their geometrical distribution. Their expressions can be obtained from

$$\frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1} = \frac{K_m - K_{m1}}{K_{m2} - K_{m1}} \quad (39)$$

and

$$\frac{\phi_3}{K_\phi} = \frac{\alpha}{K_s} - \frac{\beta_1(\alpha_1 - \phi_3)}{K_1} - \frac{\beta_2(\alpha_2 - \phi_3)}{K_2}$$
$$- (\alpha_1\beta_1 + \alpha_2\beta_2 - \alpha)\left(\frac{\alpha_1 - \alpha_2}{K_{m1} - K_{m2}}\right), \quad (40)$$

where $\alpha_i = 1 - (K_{mi}/K_i)$ and where we assume a particular class of the BM model for which the porosity of the two solid frames equals ϕ_3. We first determine the dry-rock modulus of the composite K_m as the HS average of K_{m1} and K_{m2} (equation 29), with K_i and μ_i substituted by K_{mi} and μ_{mi} in equation 30). The calculation of the bounds requires the rigidity modulus of the frames. We assume $\mu_{mi} = (\mu_i/K_i)K_{mi}$, where μ_1 is the rigidity modulus of the solid grain. In this case, the dry-rock moduli are given by

$$K_{mi} = K_i(1-\phi_3)^{A/(1-\phi_3)}, \quad i = 1, 2. \quad (41)$$

Once K_m has been obtained, we use equation 39 to calculate K_s and then use equation 38 to evaluate α, compute K_ϕ from equation 40, and finally obtain K_G from equation 36.

Fluid-saturated sandstones

Let us consider a sandstone composed of a sand matrix ($i = 1$) and a clay matrix ($i = 2$) and fully saturated with water ($i = 3$). We consider $K_1 = 37$ GPa, $\mu_1 = 44$ GPa, $K_2 = 20.8$ GPa, $\mu_2 = 6.9$ GPa, $K_3 = 2.2$ GPa, and $A = 3.5$. First, we compare our model with the BM model for varying porosity. Figure 1 shows this comparison, where the solid line is the present model, the dashed line corresponds to the BM model ($A = 3.5$), and the thin dotted line is the dry-rock modulus. The thick dotted line corresponds to the case when the clay particles are suspended in the fluid, that is, $K_{m2} = 0$

(in this case, the coefficient in equation 28 is K_{HS}/K_1 instead of K_{HS}/v). The agreement is good, although in view of topological considerations regarding the distribution of the solid phases, the BM model is not directly comparable with our model because the BM model considers separate porous matrices and our model assumes interpenetrating matrices, as does the framework model of Arns (2002). When clay is suspended in the fluid, it does not contribute to the matrix stiffness, so the corresponding modulus is lower than the other wet-rock moduli.

Gassmann's equation should predict the modulus of the saturated rock from the dry-rock modulus. In the following, we predict the wet-rock moduli obtained by Arns (2002) by fitting the corresponding dry-rock moduli and using the generalized Gassmann equation. We consider the case with 33% clay (dolomite) content ($\beta_2 = 1/3$). The data (Arns, figures 6.23a and 6.27a) correspond to the framework model (Arns, figure 6.3). The moduli of pure dolomite are $K_2 = 69.4$ GPa and $\mu_2 = 51.6$ GPa (Arns, table 6.1). Figure 2 shows the bulk moduli of clay-bearing and dolomite-bearing sandstones as a function of porosity. The solid and dashed lines correspond to the wet and dry rocks. The symbols correspond to Arns' computations with the finite-element method. Values of A equaling 3 and 2.7 are used for quartz/clay and quartz/dolomite, respectively. As can be seen, the prediction of the wet-rock moduli is very good.

Next, we use the model to fit the data set published by Han et al. (1986), obtained at a differential pressure of nearly 40 MPa. Han et al. provide measurements of the bulk density, clay content, porosity, and P- and S-wave velocities for 75 sandstone samples with porosities ranging from 2% to 30% and clay content from 0% to 50%. The experimental bulk modulus is computed from the wet-rock velocities by the expression

$$K = \rho\left(V_P^2 - \frac{4V_S^2}{3}\right), \qquad (42)$$

where ρ is the bulk density and where V_P and V_S are the P- and S-wave velocities, respectively. The fit is shown in Figure 3,

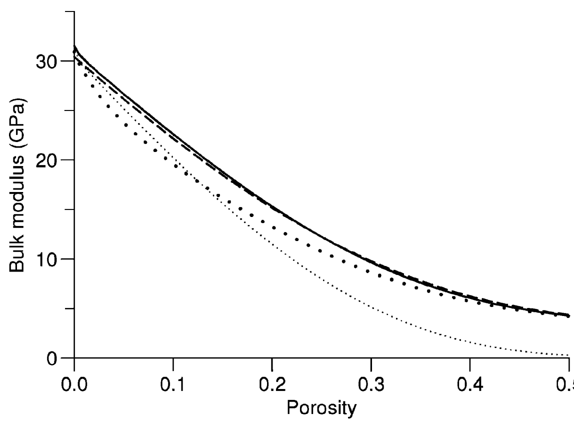

Figure 1. Comparison between the present model (solid line) and the BM model (dashed line). The thick dotted line corresponds to the case when the clay particles are suspended in the fluid; the thin dotted line is the dry-rock modulus.

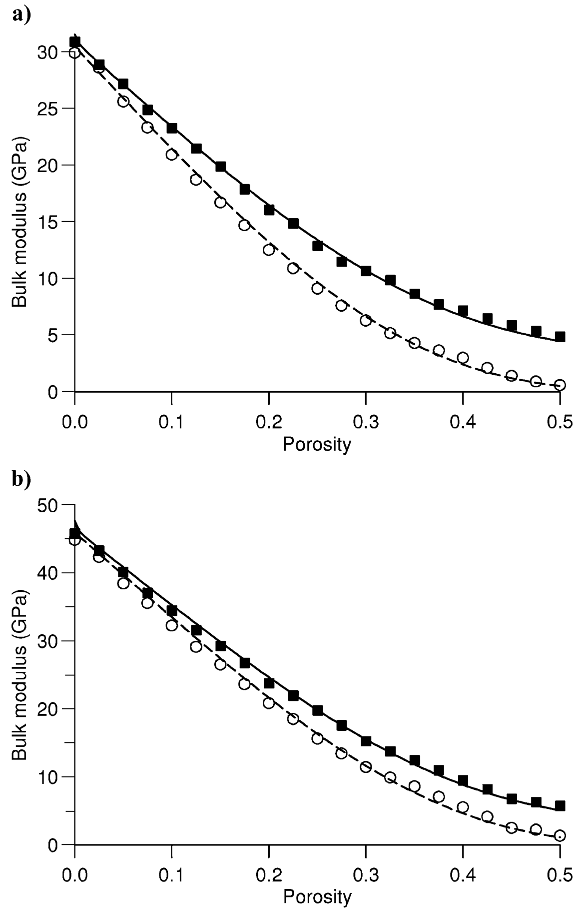

Figure 2. Bulk moduli of (a) clay-bearing and (b) dolomite-bearing sandstones as a function of porosity. The solid and dashed lines correspond to the wet and dry rocks. The symbols correspond to Arns computations with the finite-element method.

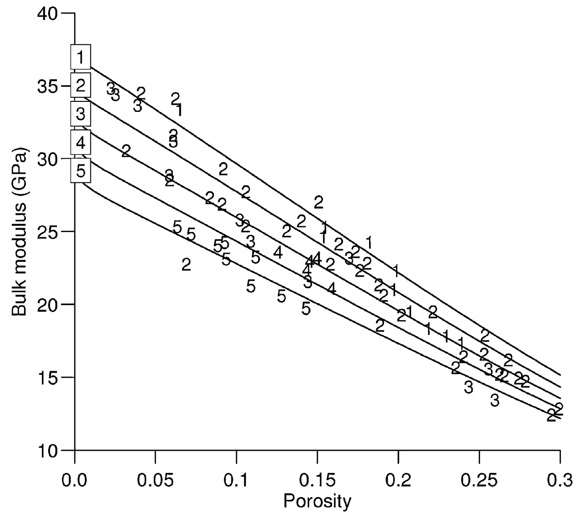

Figure 3. Bulk modulus versus porosity for different values of the clay content β_2, indicated by the numbers inside the boxes (1 = 0%, 2 = 10%, 3 = 20%, 4 = 30%, and 5 = 40%). The experimental data, represented by numbers, correspond to the data set published by Han et al. (1986). In this case, 1, 2, 3, 4, and 5 correspond to β_2 values in the ranges $[\beta_2, \beta_2 + 5\%]$, $\beta_2 = 0, \ldots, 40\%$.

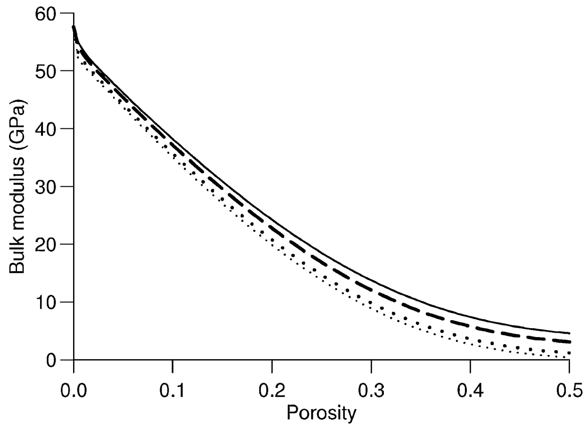

Figure 4. Bulk modulus versus porosity for a multimineral sandstone from the Barinas basin, Venezuela, saturated with different fluids: water (solid line), oil (dashed line), gas at high pressure (dotted line), and air at room conditions (thin dotted line). The bulk moduli of the fluids are 2.2, 1.4, 0.4, and 0.117 $\times 10^{-3}$ GPa, respectively.

where $A = 2.2$ and $K_2 = 18$ GPa are used. The different curves correspond to clay contents of 0%, 10%, 20%, 30%, and 50%.

Finally, we consider a calcareous sandstone from the Escandalosa Formation of the Barinas basin, Venezuela (Montoya, 2001). The rock matrix is formed by 34% quartz ($i = 1$), 28% dolomite ($i = 2$), 28% calcite ($i = 3$), and 10% clay ($i = 4$). The properties of the different minerals are $K_1 = 37.6$ GPa, $\mu_1 = 44.5$ GPa, $K_2 = 86.6$ GPa, $\mu_2 = 43.7$ GPa, $K_3 = 71.4$ GPa, $\mu_3 = 29.4$ GPa, $K_4 = 18.7$ GPa, $\mu_4 = 5.9$ GPa, and $K_5 = 2.2$ GPa (Montoya, 2001). Figure 4 shows the bulk modulus versus porosity for different saturating fluids: water (solid line), oil (dashed line), gas at high pore pressure (dotted line), and air at room conditions (thin dotted line). The bulk moduli of oil and gas are 1.4 and 0.4 GPa, respectively.

CONCLUSIONS

We have proposed a stress-strain relationship and a generalization of Gassmann's equation for a multimineral porous medium. The derivation is based on the complementary energy theorem and an analogy between the elastic coefficients of the strain-energy densities of a one-mineral porous medium (Biot's classical approach) and those of the n-mineral porous medium. The results are in good agreement with those of the Berryman-Milton model and bulk moduli computed with finite-element simulations. We have also used the model to fit experimental bulk moduli of shaley sandstones for a wide range of clay content and porosities. The analysis provides the time-domain stress-strain relation for nonuniform porosity, which can be used to simulate synthetic seismograms in heterogeneous media. Future work involves the so-called gedanken experiments, which constitute an alternative approach to obtain the elastic coefficients.

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APPENDIX A

STRAIN-ENERGY DENSITY, GENERALIZED VARIABLES AND STRESS-STRAIN RELATIONS

The purposes of this appendix are (1) to obtain the constitutive equations for nonuniform (spatially variable) porosity, which can be used to perform numerical modeling of wave propagation, and (2) to justify the expression for α_i introduced in equation 15. The variable-porosity equations for a two-phase porous medium are derived by Biot (1962), where he proposes the displacements of the matrix and the variation of fluid content as generalized coordinates. The corresponding generalized forces are the total stress components and the fluid pressure. Biot's (1962) equations correctly describe wave propagation in an inhomogeneous medium because they are consistent with Darcy's law and the boundary conditions at interfaces separating media with different properties (e.g, Gurevich and Schoenberg, 1999; Carcione, 2001).

In analogy with Biot's approach, we introduce the displacement components of the fluid relative to the solid phases (taken as a composite) as

$$w_k = \phi_n \left(u_k^{(n)} - \sum_{i=1}^{n-1} \beta_i u_k^{(i)} \right), \quad k = 1, \ldots, 3, \qquad \text{(A-1)}$$

where u denotes the macroscopic displacements and β_i is given in equation 16. The variation of fluid content is minus the divergence of the relative displacement vector defined in equation A-1. For the elementary macroscopic volume, where we assume uniform porosity ϕ_n, it is

$$\zeta = -\mathrm{div}\, w = -\phi_n \left(\theta_n - \sum_{i=1}^{n-1} \beta_i \theta_i \right), \qquad \text{(A-2)}$$

where $\theta_i = \mathrm{div}\, u^{(i)}$. The demonstration that ζ is the variation of fluid content for the composite porous medium is given below. The approach is based on the complementary energy theorem under small variations of the stresses.

The approach to evaluate the variation of strain energy and, therefore, the generalized coordinates and corresponding conjugate variables follows the development given by Carcione (2001) for a two-phase porous medium. Let us consider a volume Ω of porous material bounded by the surface S. Assume that Ω is initially in static equilibrium under the action of the surface forces — per unit volume of bulk material — acting on the different phases.

We introduce the averaged components per unit volume of solids $\tau_{kl}^{(i)}$ through the relation $\sigma_{kl}^{(i)} = \beta_i \tau_{kl}^{(i)}$, where $\sigma_{kl}^{(i)}$ are the partial stress components of phase i (see main text). These forces can be written as

$$f_k^{(i)} = \sigma_{kl}^{(i)} n_l = \beta_i \tau_{kl}^{(i)} n_l, \quad i = 1, \ldots, n-1,$$
$$f_k^{(n)} = -\phi_n p_f \delta_{kl} n_l, \qquad \text{(A-3)}$$

where $p_f = -\sigma_n/\phi_n$ is the fluid pressure, σ_n is the stress in the fluid, and n_l are the components of the outward unit vector perpendicular to S.

Assume that the system is perturbed by $\delta f_k^{(i)} (i = 1, \ldots, n)$. Let $V(\delta f_k^{(i)})$ be the strain-energy density and

$$V^* = \int_\Omega V d\Omega - \int_S \sum_{i=1}^{n} f_k^{(i)} u_k^{(i)} dS \tag{A-4}$$

be the complementary energy. Strictly, V should be the complementary strain-energy density; however, for linear stress-strain relations, V is equal to the strain-energy density (Fung, 1965). The complementary energy theorem states that of all sets of forces that satisfy the equations of equilibrium and boundary conditions, the actual one that is consistent with the prescribed displacements is obtained by minimizing the complementary energy (Fung, 1965). Then, $\delta V^* = 0$ and

$$\int_\Omega \delta V d\Omega = \int_S \sum_{i=1}^{n} \delta f_k^{(i)} u_k^{(i)} dS. \tag{A-5}$$

We have that

$$\begin{aligned} \delta f_k^{(i)} &= \beta_i \delta \tau_{kl}^{(i)} n_l, \\ \delta f_k^{(n)} &= -\phi_n \delta p_f \delta_{kl} n_l. \end{aligned} \tag{A-6}$$

Substituting these expressions into equation A-5 yields

$$\int_\Omega \delta V d\Omega = \int_S \left[\sum_{i=1}^{n-1} \beta_i \left(\delta \tau_{kl}^{(i)} - \phi_n \delta p_f \delta_{kl} \right) u_k^{(i)} - \delta p_f \delta_{kl} w_k \right] n_l dS, \tag{A-7}$$

where w_k is given in equation A-1. Applying Green's theorem to the surface integral, we obtain

$$\int_\Omega \delta V d\Omega = \int_\Omega \left[\sum_{i=1}^{n-1} \beta_i \left(\delta \tau_{kl}^{(i)} - \phi_n \delta p_f \delta_{kl} \right) u_k^{(i)} - \delta p_f \delta_{kl} w_k \right]_{,l} d\Omega, \tag{A-8}$$

where $[\,]_{,l}$ indicates the spatial derivative. Because the system is in equilibrium before and after the perturbation and the fluid pressure is constant in Ω, the stress increments must satisfy

$$\left(\delta \tau_{kl}^{(i)} \right)_{,l} = 0, \quad (\delta p_f \delta_{kl})_{,l} = 0. \tag{A-9}$$

We can write

$$\int_\Omega \delta V d\Omega = \int_\Omega \left[\sum_{i=1}^{n-1} \beta_i \left(\delta \tau_{kl}^{(i)} - \phi_n \delta p_f \delta_{kl} \right) \epsilon_{kl}^{(i)} + \zeta \delta p_f \right] d\Omega, \tag{A-10}$$

where ϵ denotes the macroscopic strains and we have used equation A-2. The symmetry of the stress tensor has been used to obtain the relation $(u_k^{(i)} \delta \tau_{kl}^{(i)})_{,l} = \epsilon_{kl}^{(i)} \delta \tau_{kl}^{(i)}$. We finally deduce from equation A-10 that

$$\delta V = \sum_{i=1}^{n-1} \beta_i \delta \left(\tau_{kl}^{(i)} - \phi_n p_f \delta_{kl} \right) \epsilon_{kl}^{(i)} + \delta p_f \zeta, \tag{A-11}$$

showing that the displacements (strains) of the solid frames and the relative fluid displacement (variation of fluid content) defined in equation A-1 (equation A-2) are the proper generalized coordinates.

The quantities $\alpha_i, i = 1, \ldots, n$ defined in equation 15 appear when the stress-strain relations (equations 18) are recast as a function of the variation of fluid content ζ. Substituting θ_n by $\sum_{i=1}^{n-1} \beta_i \theta_i - (\zeta/\phi_n)$ into equations 18 yields

$$\begin{aligned} \sigma_i &= (K_{Gi} - \phi_n \alpha_i \beta_i M)\theta_i + M(\alpha_i - \phi_n \beta_i) \\ &\quad \times \left(\sum_{m=1 (m \neq i)}^{n-1} \alpha_m \theta_m - \zeta \right), \quad i = 1, \ldots, n-1, \\ p_f &= M \left(\zeta - \sum_{m=1}^{n-1} \alpha_m \theta_m \right), \end{aligned} \tag{A-12}$$

where

$$K_{Gi} = K_{mi} + \alpha_i^2 M, \quad i = 1, \ldots, n-1. \tag{A-13}$$

Equation A-12 is a generalization to n phases of the three-phase stress-strain relation obtained by Carcione et al. (2003b), and K_{Gi} are partial Gassmann moduli. The total stress is given by

$$\sigma = \sum_{i=1}^{n-1} \sigma_i + \sigma_n = \sum_{i=1}^{n-1} \sigma_i - \phi_n p_f, \tag{A-14}$$

which, after substitution of equations A-13, becomes

$$\begin{aligned} \sigma &= \sum_{i=1}^{n-1} \left[K_{Gi} + \phi_n \alpha_i M (1 - \beta_i) + \alpha_i M \sum_{m \neq i}^{n-1} (\alpha_m - \phi_n \beta_m) \right] \\ &\quad \times \theta_i - M \left(\sum_{i=1}^{n-1} \alpha_i \right) \zeta. \end{aligned} \tag{A-15}$$

The dry-rock modulus

We can obtain an expression of the overall dry-rock modulus K_m if we assume drained conditions ($p_f = 0$) and no relative motion between the solid phases ($\theta_i \equiv \theta,\ i = 1, \ldots, n-1$). The first condition and equation A-12 imply

$$\zeta = \sum_{m=1}^{n-1} \alpha_m \theta_m. \tag{A-16}$$

Substituting this expression into the partial stress equation (A-12) gives

$$\begin{aligned} \sigma_i &= (K_{Gi} - \phi_n \alpha_i \beta_i M)\theta_i - \alpha_i M (\alpha_i - \phi_n \beta_i)\theta_i \\ &= \left(K_{Gi} - \alpha_i^2 M \right) \theta_i. \end{aligned} \tag{A-17}$$

The total stress (equation A-14) is then

$$\sigma = \sum_{i=1}^{n-1} \sigma_i = \sum_{i=1}^{n-1} \left(K_{Gi} - \alpha_i^2 M \right) \theta_i . \qquad \text{(A-18)}$$

The second condition and equation A-13 give the dry-rock or drained modulus:

$$K_m = \sum_{i=1}^{n-1} \left(K_{Gi} - M\alpha_i^2 \right) = \sum_{i=1}^{n-1} K_{mi} . \qquad \text{(A-19)}$$

Despite the similarity of equation A-19 to a Voigt average, the second condition is not that of isostrain. Let us consider the two-phase case. For isotropic strain to be the same in the fluid and solid, the physical condition is that p_1/K_1 and p_f/K_2 are the same, where $p_1 = -\sigma_1$ is the solid pressure. Pride et al. (1992) show that

$$\theta_1 = -\frac{p_1}{K_1} + \frac{d\phi}{1-\phi} \qquad \text{(A-20)}$$

and

$$\theta_2 = -\frac{p_f}{K_2} + \frac{d\phi}{\phi}, \qquad \text{(A-21)}$$

where $d\phi$ is the porosity change. Hence, $\theta_1 = \theta_2$ does not correspond to uniform strain.

The inclusion of the shear terms in the strain energy is trivial since the shear wave is not affected by the presence of the fluid.

Inclusion of shear terms

The shear invariant to include in the strain energy is $D^{(ij)} = d_{kl}^{(i)} d_{kl}^{(j)}$ (e.g., Carcione, 2001), where

$$d_{kl}^{(i)} = \epsilon_{kl}^{(i)} - \frac{1}{3}\delta_{kl}\theta_i, \quad i = 1, \ldots, n \qquad \text{(A-22)}$$

are the components of the deviatoric strain. The additional shear terms in the strain energy are

$$\sum_{i=1}^{n-1} \left(\mu_i D^{(ii)} + \sum_{j>i}^{n-1} \mu_{ij} D^{(ij)} \right), \qquad \text{(A-23)}$$

where μ_i is a shear modulus corresponding to the ith solid frame and $\mu_{ij} = \mu_{ji}$ is the interaction modulus between the ith mineral and the jth mineral, with $i \neq j$ in the second term. The expressions of the shear moduli are analogous to those of the bulk moduli (Carcione et al., 2000). Then equation A-12, including the shear terms, can be rewritten as

$$\sigma_{kl}^{(i)} = \Bigg[(K_{Gi} - \phi_n \alpha_i \beta_i M)\theta_i + M(\alpha_i - \phi_n \beta_i) \times \left(\sum_{m=1 (m \neq i)}^{n-1} \alpha_m \theta_m - \zeta \right) \Bigg] \delta_{kl} + 2\mu_i d_{kl}^{(i)} + \sum_{j \neq i}^{n-1} \mu_{ij} d_{kl}^{(j)}, \quad i = 1, \ldots, n-1. \qquad \text{(A-24)}$$

Examples of two-mineral stress-strain relations are given in Carcione et al. (2000, 2003b).

REFERENCES

Arns, C. H., 2002, The influence of morphology on physical properties of reservoir rocks: Ph.D. dissertation, University of New South Wales.

Berryman, J. G., 1995, Mixture theories for rock properties, *in* T. J. Ahrens, Ed., Handbook of Physical Constants: American Geophysical Union, 205–228.

———, 1999, Origin of Gassmann's equations: Geophysics, **64**, 1627–1629.

Berryman, J. G., and G. W. Milton, 1991, Exact results for generalized Gassmann's equation in composite porous media with two constituents: Geophysics, **56**, 1950–1960.

Biot, M. A., 1956, Theory of propagation of elastic waves in a fluid-saturated porous solid, I: Low-frequency range: Journal of the Acoustic Society of America, **28**, 168–178.

———, 1962, Mechanics of deformation and acoustic propagation in porous media: Journal of Applied Physics, **33**, 1482–1498.

Brown, R. J. S., and J. Korringa, 1975, On the dependence of the elastic properties of a porous rock on the compressibility of a pore fluid: Geophysics, **40**, 608–616.

Carcione, J. M., 2001, Wave fields in real media: Wave propagation in anisotropic, anelastic and porous media: Handbook of Geophysical Exploration, vol. 31, Pergamon Press Inc.

Carcione, J. M., and G. Seriani, 2001, Wave simulation in frozen sediments: Journal of Computational Physics, **170**, 1–20.

Carcione, J. M., F. Cavallini, J. E. Santos, C. L. Ravazzoli, and P. M. Gauzellino, 2004, Wave propagation in partially-saturated porous media: Simulation of a second slow wave: Wave Motion, **39**, 227–240.

Carcione, J. M., B. Gurevich, and F. Cavallini, 2000, A generalized Biot–Gassmann model for the acoustic properties of shaley sandstones: Geophysical Prospecting, **48**, 539–557.

Carcione, J. M., H. B. Helle, and N. H. Pham, 2003a, White's model for wave propagation in partially saturated rocks: Comparison with poroelastic numerical experiments: Geophysics, **68**, 1389–1398.

Carcione, J. M., J. E. Santos, C. L. Ravazzoli, and H. B. Helle, 2003b, Wave simulation in partially frozen porous media with fractal freezing conditions: Journal of Applied Physics, **94**, 7839–7847.

Coussy, O., 1995, Mechanics of porous continua: John Wiley & Sons.

Fung, Y. C., 1965, Solid mechanics: Prentice-Hall, Inc.

Gassmann, F., 1951, Über die elastizität poroser medium: Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, **96**, 1–23.

Goldberg, I., and B. Gurevich, 1998, A semi-empirical velocity-porosity-clay model for petrophysical interpretation of P- and S-velocities: Geophysical Prospecting, **46**, 271–285.

Gurevich, B., and J. M. Carcione, 2000, Gassmann modeling of acoustic properties of sand/clay mixtures: Pure and Applied Geophysics: **157**, 811–827.

Gurevich, B., and S. L. Lopatnikov, 1995, Velocity and attenuation of elastic waves in finely layered porous rocks: Geophysical Journal International, **121**, 933–947.

Gurevich, B., and M. Schoenberg, 1999, Interface boundary conditions for Biot's equations of poroelasticity: Journal of the Acoustic Society of America, **105**, 2585–2589.

Han, D. H., A. Nur, and D. Morgan, 1986, Effects of porosity and clay content on wave velocities in sandstones: Geophysics, **51**, 2093–2107.

Hashin, Z., and S. Shtrikman, 1963, A variational approach to the theory of the elastic behaviour of multiphase materials: Journal of the Mechanics and Physics of Solids, **11**, 127–140.

Krief, M., J. Garat, J. Stellingwerff, and J. Ventre, 1990, A petrophysical interpretation using the velocities of P and S waves (full waveform sonic): The Log Analyst, **31**, 355–369.

Leclaire, P., 1992, Propagation acoustique dans les milieux poreux soumis au gel—Modélisation et expérience: Ph.D. dissertation, Paris University.

Leclaire, P., F. Cohen-Ténoudji, and J. Aguirre-Puente, 1994, Extension of Biot's theory of wave propagation to frozen porous media: Journal of the Acoustic Society of America, **96**, 3753–3768.

———, 1995, Observation of two longitudinal and two transverse waves in a frozen porous medium: Journal of the Acoustic Society of America, **97**, 2052–2055.

Mavko, G., T. Mukerji, and J. Dvorkin, 1998, The rock physics handbook: Tools for seismic analysis in porous media: Cambridge University Press.
Montoya, N., 2001, Aplicación de resonancia magnética nuclear y ultrasonido a muestras de núcleo del pozo BOR-5E miembro O. Formación Escandalosa, Barinas: Master's thesis, Universidad Central de Venezuela.
Nolen-Hoeksema, R. C., 2000, Modulus-porosity relations, Gassmann's equations, and the low-frequency elastic-wave response to fluids: Geophysics, **65**, 1355–1361.
Pride, S. R., A. F. Gangi, and F. D. Morgan, 1992, Deriving the equations of motion for porous isotropic media: Journal of the Acoustic Society of America, **92**, 3278–3290.
Roberts, A. P., and A. J. Garboczi, 2000, Elastic properties of model porous ceramics: Journal of the American Ceramic Society, **83**, 3041–3048.
Santos, J. E., J. Douglas Jr., and J. Corberó, 1990, Static and dynamic behaviour of a porous solid saturated by a two-phase fluid: Journal of the Acoustic Society of America, **87**, 1428–1438.
Tserkovnyak, Y., and D. L. Johnson, 2003, Capillary forces in the acoustics of patchy-saturated porous media: Journal of the Acoustic Society of America, **114**, 2596–2606.
White, J. E., 1975, Computed seismic speeds and attenuation in rocks with partial gas saturation: Geophysics, **40**, 224–232.