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PHYSICAL ACOUSTICS

Quality Factor of Inhomogeneous Plane Waves

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Abstract—We first compare two apparently dissimilar expressions for the quality factor Q of inhomogeneous plane waves in isotropic viscoelastic media, where Q is defined as the result of an energy-balance relation. In this case, it is the ratio of twice the strain energy to the dissipated energy. Both expressions give the same Q for P-waves and Q for S-waves is independent of the inhomogeneity angle γ (isotropic media). Then, we consider the more general balance equation, which holds for anisotropic viscoelastic media, where anomalous behaviors are observed when γ exceeds some critical value (forbidden directions of propagation). This problem has already been solved analytically for SH-waves. Here, we consider the qP–qS case, which requires a numerical solution of the dispersion equation to obtain the wavenumber and attenuation factor, i.e., the real and imaginary parts of the wave vector, respectively. The forbidden directions appear when the phase velocity approaches a zero value. Generally, the phase velocity of homogeneous waves ($\gamma = 0$) exceeds that of inhomogeneous waves, while the latter show stronger attenuation (lower quality factor). In the vicinity of the forbidden directions, the opposite behavior may occur.

Keywords: quality factor, anisotropic viscoelastic medium, energy-balance relation, attenuation factor

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1. INTRODUCTION

There are several works in the literature dealing with anisotropy and viscoelasticity on various aspects. For instance, Zaĭtsev and Kuznetsova [1] and Zaĭtsev et al. [2] analyze the energy density and power flow of waves propagating in unbounded piezoelectric media and provide an alternative definition of potential energy, respectively. Djeran-Maigre and Kuznetsov [3] study wave dispersion in plates. These works consider anisotropy but not intrinsic absorption (viscoelasticity). On the other hand, Moradi and Innanen [4] analyze the reflection of inhomogeneous plane waves in viscoelastic media, whereas Morozov et al. [5] and Liu et al. [6] consider the effects viscoelasticity on wave propagation and how to approximate a constant Q with Zener mechanical models, respectively.

Generally, plane waves in anelastic media are inhomogeneous, i.e., equiphase planes do not coincide with equi-amplitude planes, which means that the attenuation vector does not point in the same direction as the wave vector) (Buchen [7], Carcione [8]). There is a distinct difference between the inhomogeneous waves of lossless media (interface or guided waves) and those of viscoelastic media (body waves). In the former case, the direction of attenuation is normal to the direction of propagation, whereas for inhomogeneous viscoelastic waves that angle must be less than $\pi/2$, and this is valid in 2D and 3D spaces. Furthermore, for viscoelastic inhomogeneous waves the energy does not propagate in the direction of the slowness vector and the particle motion is elliptical in general (Carcione [8]).

There are two apparent dissimilar expressions for the quality factor Q reported in Carcione [8]. Eqs. (3.133) in this book summarize the average energies corresponding to the P- and S-waves, used to define the quality factors. These equations are based on the work of Buchen [7] and hold for isotropic media. On the other hand, Eq. (4.198) in Carcione [8] provides an expression of the quality factor for anisotropic and viscoelastic media obtained from a general energy balance developed by Carcione and Cavallini [9]. For completeness, all the equations related to this expression are given in Appendix A. Since it is not clear if the two approaches give the same quality factor for isotropic media, we compare the two expressions for Q as a function of the inhomogeneity angle.

In addition, we study the anisotropic case for qP–qS waves in the symmetry plane of an orthorhombic medium. In a medium with anisotropy and attenuation, beyond a given degree of wave inhomogeneity, the theory predicts forbidden directions (forbidden solutions) (Carcione and Cavallini [10, 11]). Červený and Pšenčík [12] have used a form of the sextic Stroh formalism to re-interpret the forbidden-directions phenomenon by using a different inhomogeneity parameter, instead of the angle between the propagation and attenuation directions

2. THEORY

2.1. Expressions for the Quality Factors

Following the notation of Buchen [7] and Chapter 3 of Carcione [8], we consider the viscoelastic plane-wave solution of the particle velocity

$$\mathbf{v} = \mathbf{V}\exp\left[i\left(\omega t - \mathbf{k}\cdot\mathbf{x}\right)\right], \tag{1}$$

where t is time, $\mathbf{x}$ is the position vector, ω is the angular frequency, $\mathbf{k} = \omega\mathbf{s}$ is the wave vector, $\mathbf{s}$ is the slowness vector, $\mathbf{V}$ is a complex vector and $i = \sqrt{-1}$. The relevant quantity is the complex wave vector

$$\mathbf{k} = \boldsymbol{\kappa} - i\boldsymbol{\alpha} = \kappa\hat{\boldsymbol{\kappa}} - i\alpha\hat{\boldsymbol{\alpha}}, \tag{2}$$

with $\boldsymbol{\kappa}$ being the real wave vector and $\boldsymbol{\alpha}$ the attenuation vector. They express the magnitudes of both the wavenumber κ and the attenuation factor α, and the directions of the normals to planes of constant phase and planes of constant amplitude (vectors with a hat are unit vectors). If γ is the (inhomogeneity) angle between $\boldsymbol{\kappa}$ and $\boldsymbol{\alpha}$, the wave is homogeneous for $\gamma = 0$, i.e, $\mathbf{k} = (\kappa - i\alpha)\,\hat{\boldsymbol{\kappa}} \equiv k\hat{\boldsymbol{\kappa}}$.

Let us consider the quality factor as given by Buchen [7]. The rate of energy dissipated in one cycle is

$$\begin{aligned}\langle\dot{D}\rangle &= -\frac{1}{2}\omega|\Phi_0|^2\exp(-2\boldsymbol{\alpha}\cdot\mathbf{x})\,\mathrm{Im}\left(k^2\right)\\ &\times\left[\rho\omega^2 - 2\mu_I\,\mathrm{Im}\left(k^2\right)\tan^2\gamma\right],\end{aligned} \tag{3}$$

or (since $\langle\dot{D}\rangle = \omega\langle D\rangle$), the dissipated energy is

$$\begin{aligned}\langle D\rangle &= \frac{1}{2}|\Phi_0|^2\exp(-2\boldsymbol{\alpha}\cdot\mathbf{x})\,\mathrm{Im}\left(k^2\right)\\ &\times\left[-\rho\omega^2 + 2\mu_I\,\mathrm{Im}\left(k^2\right)\tan^2\gamma\right],\end{aligned} \tag{4}$$

where Φ_0 is a constant amplitude. An alternative option to avoid real and imaginary parts in these expressions is to use complex conjugate quantities (e.g., Djeran-Maigre and Kuznetsov [3]). There are some sign differences between the expressions in Buchen [7]) and Carcione [8]. Buchen defines $G = G_1 - iG_2 = \mu_R + i\mu_I$ (the complex shear modulus) and uses the opposite convention for the time Fourier transform so that $\omega \to -\omega$. Also, Buchen defines the complex wave vector as $\kappa + i\alpha$ and not as $\kappa - i\alpha$ (more precisely, Eq. (3.94) in Carcione [8] is the same as Eq. (36) in Buchen [7], but Eq. (3.98) in Carcione [8], if obtained from Eq. (35) of Buchen [7], leads to equation (3)).

Since

$$\mathrm{Im}\left(k^2\right) = -2\boldsymbol{\kappa}\cdot\boldsymbol{\alpha}, \tag{5}$$

and

$$\left[\mathrm{Im}\left(k^2\right)\tan\gamma\right]^2 = 4\kappa^2\alpha^2\sin^2\gamma = 4|\boldsymbol{\kappa}\times\boldsymbol{\alpha}|^2, \tag{6}$$

we have

$$\langle D\rangle = |\Phi_0|^2\exp(-2\boldsymbol{\alpha}\cdot\mathbf{x})\left[\rho\omega^2(\boldsymbol{\kappa}\cdot\boldsymbol{\alpha}) + 4\mu_I|\boldsymbol{\kappa}\times\boldsymbol{\alpha}|^2\right]. \tag{7}$$

Similarly, Eqs. (3.94) and (3.97) of Carcione [8] give the averaged strain energy,

$$\begin{aligned}\langle V\rangle &= \frac{1}{4}|\Phi_0|^2\exp(-2\boldsymbol{\alpha}\cdot\mathbf{x})\\ &\times\left\{\rho\omega^2\,\mathrm{Re}\left(k^2\right) + 2\mu_R\left[\mathrm{Im}\left(k^2\right)\tan\gamma\right]^2\right\},\end{aligned} \tag{8}$$

or

$$\begin{aligned}\langle V\rangle &= \frac{1}{4}|\Phi_0|^2\exp(-2\boldsymbol{\alpha}\cdot\mathbf{x})\\ &\times\left[\rho\omega^2\left(|\boldsymbol{\kappa}|^2 - |\boldsymbol{\alpha}|^2\right) + 8\mu_R|\boldsymbol{\kappa}\times\boldsymbol{\alpha}|^2\right],\end{aligned} \tag{9}$$

which coincides with that of Buchen [7]. The Q factor qP or qS waves is then

$$Q_B = \frac{2\langle V\rangle}{\langle D\rangle} = \frac{\rho\omega^2\,\mathrm{Re}\left(k^2\right) + 2\mu_R\left[\mathrm{Im}\left(k^2\right)\tan\gamma\right]^2}{\mathrm{Im}\left(k^2\right)\left[-\rho\omega^2 + 2\mu_I\,\mathrm{Im}\left(k^2\right)\tan^2\gamma\right]}. \tag{10}$$

It is shown in Appendix B that Q for qS waves does not depend on γ.

On the other hand, the energy balance obtained by Carcione and Cavallini [9] yields

$$Q_{CV} = -\frac{\mathrm{Re}\left[(\beta^* X + \xi^* W)k_1^* + (\beta^* W + \xi^* Z)k_3^*\right]}{2\left[\mathrm{Re}(\beta^* X + \xi^* W)\,\mathrm{Im}(k_1) + \mathrm{Re}(\beta^* W + \xi^* Z)\,\mathrm{Im}(k_3)\right]}, \tag{11}$$

where the quantities involved in this expression are given in Appendix A (the superscript "∗" denotes complex conjugate). This quality factor is more general than Q_B, since it holds for anisotropic media (see Carcione [8], Eq. (4.198)].

2.2. Wavenumber Components

The wavenumber components can be written as

$$k_i = \omega s_i = \kappa l_i - i\alpha m_i, \quad i = 1,\ldots,3, \tag{12}$$

where $\hat{\boldsymbol{\kappa}} = (l_1, l_3) = (\sin\theta, \cos\theta)$ and $\hat{\boldsymbol{\alpha}} = (m_1, m_3) = (\sin(\theta+\gamma), \cos(\theta+\gamma))$ are unit vectors in equation (2) defining the propagation and attenuation directions, with θ the propagation angle.

2.2.1. Isotropic case. In the isotropic case, we simply have

$$v_c(P \text{ wave}) = \frac{\omega}{k_P} = \sqrt{\frac{p_{11}}{\rho}}, \quad \text{and}$$
$$v_c(S \text{ wave}) = \frac{\omega}{k_S} = \sqrt{\frac{p_{44}}{\rho}}, \tag{13}$$

where p_{11} and p_{44} are the P- and S-wave complex and frequency-dependent stiffnesses, with

$$2\kappa^2 = \mathrm{Re}(k^2) + \sqrt{[\mathrm{Re}(k^2)]^2 + [\mathrm{Im}(k^2)]^2 \sec^2\gamma},$$
$$2\alpha^2 = -\mathrm{Re}(k^2) + \sqrt{[\mathrm{Re}(k^2)]^2 + [\mathrm{Im}(k^2)]^2 \sec^2\gamma}, \tag{14}$$

(see Eqs (3.34) in Carcione [8]].

2.2.2. Anisotropic case. The procedure to obtain the equations in this case follows that of the SH-wave (Carcione and Cavallini [10, 11]), but the problem has to be solved numerically (Carcione and Ursin [13]). The dispersion relation for qP–qS waves (leading to equation (A.1)) is

$$D \equiv (p_{11}k_1^2 + p_{44}k_3^2 - \rho\omega^2)(p_{33}k_3^2 + p_{44}k_1^2 - \rho\omega^2) - (p_{13} + p_{44})^2 k_1^2 k_3^2 = 0. \tag{15}$$

We then use equation (12) and solve for κ and α from

$$\mathrm{Re}[D(\kappa,\alpha)] = 0, \quad \mathrm{Im}[D(\kappa,\alpha)] = 0. \tag{16}$$

Equations (16) are solved using the Newton–Raphson method for a nonlinear systems of equations.

The phase velocity and attenuation factors are

$$v_p = \frac{\omega}{\kappa}, \quad \text{and } \alpha = -\mathrm{Im}(k), \tag{17}$$

respectively.

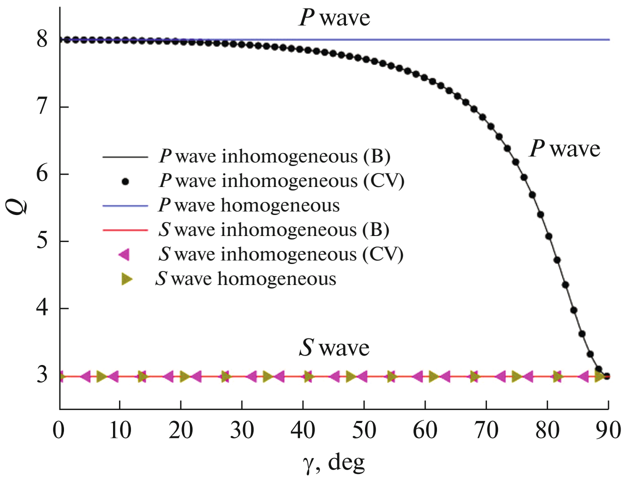

Fig. 1. Quality factor as a function of the inhomogeneity angle, based on Buchen's energy balance [7] (B) and Carcione and Cavallini's energy balance [9] (CV).

3. EXAMPLES

Let us first consider an isotropic medium defined by

$$p_{11} = (40 + 5i)\ \mathrm{GPa},$$
$$p_{44} = (21 + 7i)\ \mathrm{GPa},$$
$$p_{33} = p_{11},$$
$$p_{13} = p_{11} - 2p_{44},$$

and a density $\rho = 2500$ kg/m^3.

We define the quality factors

$$Q_{IJ} = \frac{\mathrm{Re}(p_{IJ})}{\mathrm{Im}(p_{IJ})}. \tag{18}$$

For the above medium, the quality factors of homogeneous waves are $Q_P = Q_{11} = 8$ and $Q_S = Q_{44} = 3$. Figure 1 shows the quality factors, where the solid black line corresponds to Q_B and the solid dots to Q_{CV}. In isotropic media, any value of θ yields the same result, and $\rho\omega^2 = p_{II}k^2$, independent of frequency in this example ($II = 11$ for P-waves and $II = 44$ for qS-waves). As it can be seen, the two approaches are equivalent, with the S-wave Q factors independent of γ. Since $\alpha = -\mathrm{Im}(k)$, and k does not depend on γ, the attenuation factor is also independent of the inhomogeneity angle. It can be shown that the S-wave quality factor for inhomogeneous waves is also independent of the inhomogeneity angle in the isotropic poro-viscoelastic case. Equation (7.613) in Carcione [8] gives the general quality factor for qP- and qS-waves in the anisotropic poro-viscoelastic case, and the corresponding energy balance is given in Carcione [14].

The advantage of the energy balance developed by Carcione and Cavallini [9] is that it holds for anisotropic viscoelastic media (see also Carcione and Cavallini [10, 11]), and poro-viscoelastic anisotropic media

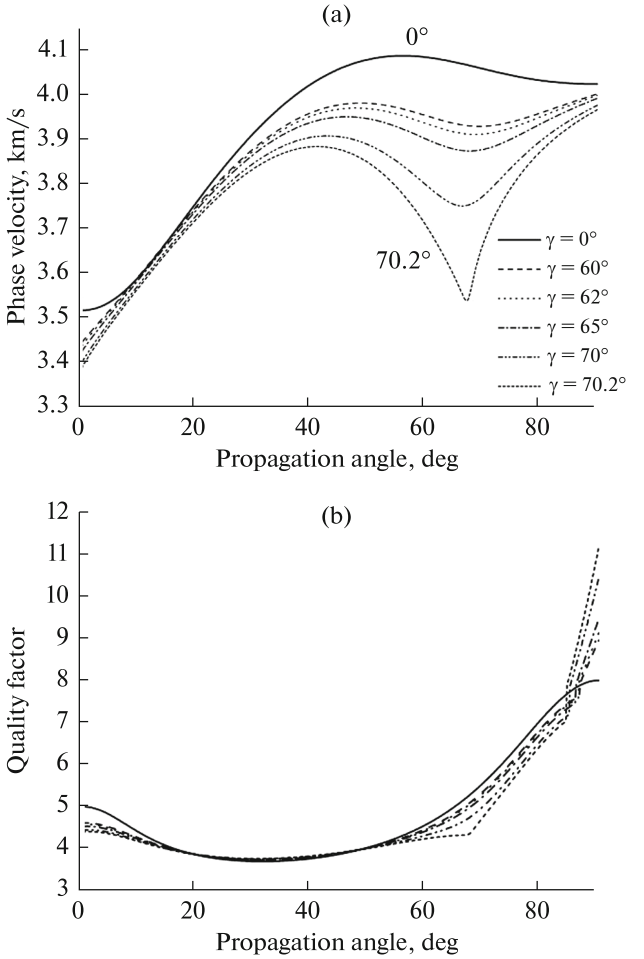

Fig. 2. (a)—Phase velocity and (b)—quality factor of the *qP* waves as a function of the propagation angle for different values of the inhomogeneity angle, based on Carcione and Cavallini's energy balance [9]. The medium is anisotropic.

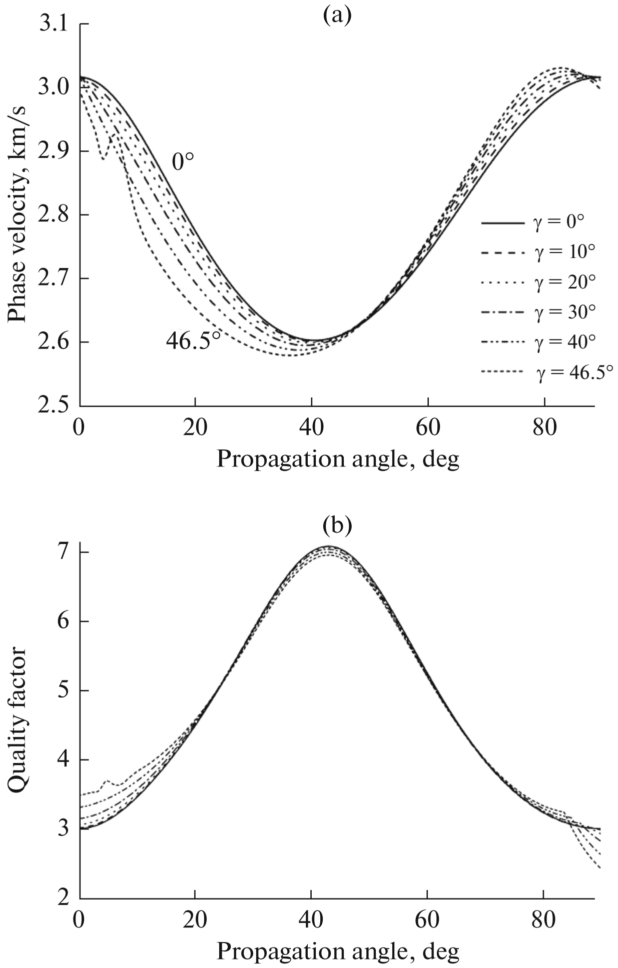

Fig. 3. (a)—Phase velocity and (b)—quality factor of the *qS* waves as a function of the propagation angle for different values of the inhomogeneity angle, based on Carcione and Cavallini's energy balance [9]. The medium is anisotropic.

(Carcione [14]). We now consider the anisotropic viscoelastic case, with the following components:

$$p_{11} = (40 + 5i)\ \text{GPa},\quad Q_{11} = 8,$$
$$p_{44} = (21 + 7i)\ \text{GPa},\quad Q_{44} = 3,$$
$$p_{33} = (30 + 6i)\ \text{GPa},\quad Q_{33} = 5,$$
$$p_{13} = (1 + i)\ \text{GPa}.$$

Figures 2 and 3 shows the phase velocity (Fig. 1a) and quality factor (Fig. 1b) (Q_{CV}) of the *qP*- and *qS*-waves as a function of the propagation angle for different values of the inhomogeneity angle. The last five γ values are close to the critical inhomogeneity angle, where the forbidden directions start to appear and the curves show an anomalous behavior, because the phase velocity is decreasing remarkably (Carcione and Cavallini [10]). For *P*-waves, the anomalous propagation angle is approximately $\theta = 70°$, while for the *S*-waves this behavior appears between 5° and 10°. The respective critical inhomogeneities angles are approximately 70.2° and 46.5°, respectively. The critical angle depends on the propagation angle θ and the medium properties, p_{IJ} and a precise evaluation requires a numerical solution of a complex-valued sixth-degree polynomial equation (Červený and Pšenčík [15]). The cause of these anomalous behaviors in the curves of Figs. 2a and 3a at high inhomogeneity angles and specific propagation angles is a combined effect of anisotropy and viscoelasticity. Contrary to the isotropic case, the quality factor of the *S*-wave is γ-dependent. This dependence of attenuation on the inhomogeneity angle can clearly be seen in the attenuation factors (α) shown in Fig. 4, where we have assumed a frequency of 50 Hz. In general, the phase velocity of homogeneous waves is greater than that of inhomogeneous waves, while the latter are more attenuated (see Fig. 4). Clearly, the critical inhomogeneity angle is different for *P*- and *S*-waves.

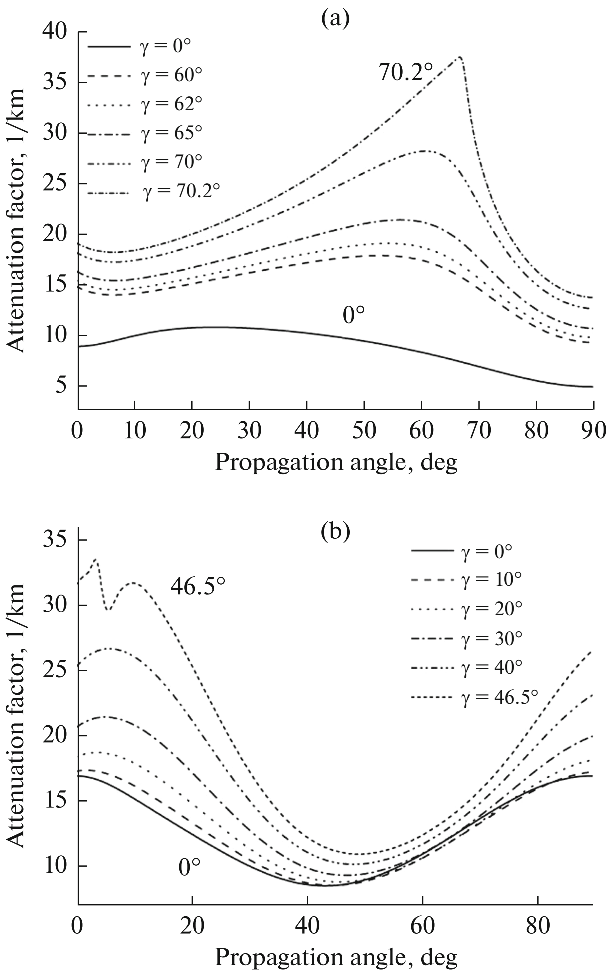

Fig. 4. Attenuation factor of the (a)—*qP*- and (b)—*qS*-waves as a function of the propagation angle for different values of the inhomogeneity angle, based on Carcione and Cavallini's energy balance [9]. The medium is anisotropic.

4. CONCLUSIONS

We have shown that for plane inhomogeneous *S*-waves in isotropic media, the ratio of twice the average strain energy to the average dissipated energy are equal and independent of the degree of inhomogeneity of the waves. This has been verified for two apparent dissimilar energy balances, which yield the same *P*-wave quality factor as a function of the inhomogeneity angle, ranging from the homogeneous *Q* (*P*) value at 0 degrees to the homogeneous *Q* (*S*) value at 90 degrees, so that the attenuation related to the *S*-waves constrains that of the *P* waves. In anisotropic media, all the properties depend on the inhomogeneity of the waves, with the phase velocity of homogeneous waves showing higher values, and the attenuation weaker values compared to that of the inhomogeneous waves. This behavior may not be verified in the vicinity of forbidden directions, where anomalous behaviors occur at certain propagation angles.

This work clarifies the fact that the two energy balances are equivalent, and while the first (older) approach by Buchen holds only for isotropic media, the second is valid for anisotropic viscoelastic media.

APPENDIX A

Quality Factor for qP–qS Waves in Anisotropic Viscoelastic Media

We report an expression of the quality factor for anisotropic and viscoelastic media based on the energy balance obtained by Carcione and Cavallini [9]. We assume wave propagation in the (x, z)-symmetry plane of an orthorhombic medium, with the positive z-axis pointing downwards. The medium complex and frequency-dependent stiffness components are p_{IJ}, $I, J = 1, \ldots, 6$ and the mass density is ρ. We consider the general plane-wave solution (1), where the wave vector is $\mathbf{k} = (k_1, k_3) = \omega(s_1, s_3)$, where s_i are slowness components. The k_3-component in terms of the horizontal wavenumber k_1 is

$$k_3 = \pm\frac{\omega}{\sqrt{2}}\sqrt{K_1 \mp \mathrm{pv}\sqrt{K_1^2 - 4K_2K_3}}, \tag{A.1}$$

where

$$K_1 = \rho\left(\frac{1}{p_{55}} + \frac{1}{p_{33}}\right) + \frac{1}{p_{55}}\left[\frac{p_{13}}{p_{33}}(p_{13} + 2p_{55}) - p_{11}\right]s_1^2,$$

$$K_2 = \frac{1}{p_{33}}\left(p_{11}s_1^2 - \rho\right), \quad K_3 = s_1^2 - \frac{\rho}{p_{55}}.$$

The signs in k_3 correspond to $(+, -)$: downward propagating *qP* wave; $(+, +)$: downward propagating *qS* wave; $(-, -)$: upward propagating *qP* wave; and $(-, +)$: upward propagating *qS* wave. The plane-wave eigenvectors (polarizations) are

$$\mathbf{V} = V_0\begin{pmatrix}\beta\\ \xi\end{pmatrix}, \tag{A.2}$$

where V_0 is the plane-wave amplitude and

$$\beta = \mathrm{pv}\sqrt{\frac{p_{55}k_1^2 + p_{33}k_3^2 - \rho\omega^2}{p_{11}k_1^2 + p_{33}k_3^2 + p_{55}\left(k_1^2 + k_3^2\right) - 2\rho\omega^2}}, \tag{A.3}$$

and

$$\xi = \pm\,\mathrm{pv}\sqrt{\frac{p_{11}k_1^2 + p_{55}k_3^2 - \rho\omega^2}{p_{11}k_1^2 + p_{33}k_3^2 + p_{55}\left(k_1^2 + k_3^2\right) - 2\rho\omega^2}}. \tag{A.4}$$

In general, the + and − signs correspond to the *qP*- and *qS*-waves, respectively. However one must choose the signs such that ξ varies smoothly with the propagation angle.

Then, the quality factor for anisotropic media is given by equation (11), where

$$\omega W = p_{55}(\xi k_1 + \beta k_3), \quad \omega X = \beta p_{11} k_1 + \xi p_{13} k_3, \quad \omega Z = \beta p_{13} k_1 + \xi p_{33} k_3. \tag{A.5}$$

In the isotropic case the above components k_1 and k_3 are equivalent to equation (12). In the anisotropic case, a numerical solution is required to obtain k_1 and k_3, i.e., the calculation of κ and α, but in this case, equation (A.1) is the analytical solution for homogeneous waves.

APPENDIX B

Quality Factor of S-waves for Inhomogeneous Waves

For S-waves (isotropic media),

$$\frac{\rho\omega^2}{\mu} = \frac{k^2}{\omega^2} = \frac{\mathrm{Re}(k^2) + i\,\mathrm{Im}(k^2)}{\omega^2}. \tag{B.1}$$

Since $\mu = \mu_R + i\mu_I$ and taking real and imaginary parts, we obtain

$$\rho\omega^2 = \mu_R \mathrm{Re}(k^2) - \mu_I \mathrm{Im}(k^2), \quad 0 = \mu_R \mathrm{Im}(k^2) - \mu_I \mathrm{Re}(k^2). \tag{B.2}$$

Substituting equations (B.2) into equation (10), we obtain

$$Q = \frac{2\langle V\rangle}{\langle D\rangle} = -\frac{\mathrm{Re}(k^2)}{\mathrm{Im}(k^2)}. \tag{B.3}$$

which is the S-wave Q factor of homogeneous plane waves, independent of the inhomogeneity angle γ (see Eqs. (3.32) in Carcione [8]). This result is also valid for SH-waves.

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