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## Rock acoustics of CO<sub>2</sub> storage in basalt

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### SUMMARY

We simulate variations in the P- and S-wave velocities due to  $CO_2$  injection in basalt, caused by water replacement and mineralization in the form of carbonate cements. Carbonization can be monitored with seismic data on the basis of changes in velocity. We consider basalts composed of (Fe, Ca and Mg) minerals mainly, and the pore space is assumed to be initially saturated with water. The mineralization is based on a model that estimates the volume of precipitated cement and resulting porosity loss, and the velocities as a function of injection time are calculated with a patchy cement model. In the example, there are two stages: 1 week of  $CO_2$  injection and 29 weeks of frame cementation. After 1 week, water has been replaced by  $CO_2$ , where density and P-wave velocity decrease, and cementation implies an increase in bulk density, P-wave velocity, whereas porosity and surface area are reduced. To our knowledge there are no models relating the effects of carbon mineralization on seismic properties. The present model can be applied to other rocks, where  $CO_2$  injection induces mineralization and storage.

**Key words:** Composition and structure of the continental crust; Microstructure; Acoustic properties; Wave propagation.

#### **1 INTRODUCTION**

Storage of carbon dioxide (CO<sub>2</sub>) in volcanic rocks, such as basalt, peridotite and tuffaceous rocks, rich in reactive and divalent cations, is a valid approach for reducing the amount of greenhouse gases in the atmosphere (e.g. Lackner *et al.* 1995; Takaya *et al.* 2015; Wu *et al.* 2021).

The southwestern coastal region of Saudi Arabia has a dense sequence of Oligocene basalts in the Jizan Group, which have accumulated along the continental rift which preceded the opening of the Red Sea. These basalts are intended for disposal of  $CO_2$  emitted from industrial sources due to carbon mineralization in the subsoil processes. Oelkers *et al.* (2022) reviewed the geology and geochemistry of the basalts of the Jizan Group and evaluated the reactivity of representative samples. Their results assess the fate of dissolved  $CO_2$  in water injected into the Jizan Group. The  $CO_2$ -charged water will carbonate when interacting with the basaltic rocks, and would fix more than 95 per cent of injected water-dissolved  $CO_2$  within 5 yrs at 100 °C. Fedorik *et al.* (2023) state that the Jizan Group is one of the principal  $CO_2$  sinks that can help fulfil the decarbonization expectations of the country.

Basalts react with a  $CO_2$ -water mixture, resulting in the precipitation of minerals. In particular, Adam *et al.* (2013) reacted three ferro-basaltic samples from the Snake River Plain

with a water– $CO_2$  mixture. They observed that after 30 weeks carbonates precipitated in the pore space, within grain contacts and microcracks reducing the rock permeability and stiffening the rock.  $CO_2$  injection in basalts can increase or decrease the wave velocities. Actually, a small amount (<5 per cent vol.) of carbonate precipitation (Adam *et al.* 2013) or glass dissolution (Kanakiya *et al.* 2017; Saxena *et al.* 2018) results in a change in P-wave velocity of up to 22 and -14 per cent, respectively, for short-term experiments. As for the experimental data, we used those of L. Adam and coworkers, who were the only research group to study this problem. However, in fig. 1 of Otheim *et al.* (2011), only a schematic representation of the process is shown.

The mineralization process is similar to a diagenesis one, where minerals precipitate from solution and cement grain contacts and fill cracks, with a reduction in porosity (Pytte & Reynolds 1989; Walderhaug 1996; Lander & Walderhaug 1999). The acoustics of mineralization and cementation has scarcely been attacked from a theoretical point of view. Khatiwada *et al.* (2012) performed a feasibility study of time-lapse seismic monitoring of CO<sub>2</sub> sequestration in a basalt reservoir. They computed wave propagation simulations, but the combined fluid substitution (CO<sub>2</sub> replaces water) and mineralization effects on the elastic properties are shown schematically (qualitatively). The mineralization approach starts by a computation of the volume of carbonate cement precipitated in the pore volume by using the Walderhaug (1996) model, which gives the fraction of cement, porosity and surface area of the pore space as a function of the injection time. According to Walderhaug (1996), the first stage is precipitation and then cementation starts to be effective. The process depends on the effective radii of the grains (and surface area), with small grains producing more cement. Roughly, the porosity loss is equal to the volume of precipitated carbonates (Lander & Walderhaug 1999).

A petro-elastical model yields the seismic velocities of the rock. The approach is based on the contact-cement theory (CCT) of Dvorkin & Nur (1996), assuming cement deposited at grain contacts (scheme 1 in Mavko *et al.* 2020, p. 355), and the Hertz-Mindlin (HM) theory (see also Avseth *et al.* 2010, 2016). Then, the Voigt-Reuss-Hill averages and Gassmann equation yield the wetrock moduli, with the self-consistent (SC) model used to obtain the properties of the mineral mixture (Dvorkin *et al.* 1999).

#### 2 THEORY

#### 2.1 Mineral cement precipitation

Basalts are rich in iron, magnesium and calcium, with a minor percentage of silica (quartz). Basalts react with a  $CO_2$ -water mixture, resulting in the precipitation of minerals (Ca, Mg and Fe cations) between adjacent grains and cracks and microfractures, which are assumed to contribute as grain contacts in the proposed acoustic theory given in the next section. To our knowledge, there is no theory to model the effects of cement in fractures on P-wave velocity. The first reaction is the formation of carbonic acid:

 $CO_2 + H_2O \rightarrow H_2CO_3$  (carbonic acid)

Then, those cations precipitate as carbonates cements:

(Ca, Mg, Fe) + H<sub>2</sub>CO<sub>3</sub>  $\rightarrow$  (Ca, Mg, Fe)CO<sub>3</sub> (carbonates) + 2H Walderhaug (1996) obtained the amount of quartz cement precipitated in a sandstone as a function of time. We use his equations as an *ad hoc* phenomenological approach applied to the precipitated of metal minerals in basalt. The amount of cement (m<sup>3</sup>) precipitated in a 1 m<sup>3</sup> of basalt at time  $t_{l+1}$  is

$$\phi_p^{l+1} = \phi_0 - \left(\phi_0 - \phi_p^l\right) \exp\left[-\frac{aMA_0}{\rho_1\phi_0} \left(10^{bt_{l+1}} - 10^{bt_l}\right)\right],\tag{1}$$

where  $\phi_p^l$  is the amount of mineral cement present at time  $t_l$ ,  $\phi_0$  is the initial porosity, *a* and *b* are constants, which have units of mol m<sup>-5</sup> and s<sup>-1</sup>, respectively,  $\rho_1$  is the grain density, *M* [kg mol<sup>-1</sup>] is the molar mass of mineral and is the initial mineral surface area,

$$A_0 = \frac{6FV}{D},\tag{2}$$

where D is the grain diameter, F is the fraction of mineral in the rock compared to silica and V is a unit volume (1  $m^3$  if D is given in metres).

In the first time step,  $t_0 = 0$ ,  $\phi_p^0 = 0$  and

$$\frac{\phi_p^1}{\phi_0} = 1 - \exp\left[-\frac{aMA_0}{\rho_1\phi_0}(10^{bt_1} - 1)\right], \quad t_1 = dt,$$
(3)

where dt is the time step.

The porosity varies as

$$\phi = \phi_0 - \phi_p \qquad \left(\phi^l = \phi_0 - \phi_p^l\right) \tag{4}$$



Figure 1. Simplified scheme showing basalt after cementation, where  $\phi_p$  is the cement fraction bonded to the grains. Water is absent after complete CO<sub>2</sub> injection, where cementation initiates.

and the surface area as

$$A = A_0 \frac{\phi}{\phi_0}.$$
 (5)

#### 2.2 Acoustic patchy-cement model

We consider a petro-elastical model to obtain the P- and S-wave velocities as a function of injection time. For the equations regarding the Voigt, Reuss and Hashin–Shtrikman (HS) averages, we refer to Mavko *et al.* (2020). Dvorkin & Nur (1996) developed an elastic model, called contact cement theory (CCT), considering spherical grains (see also Avseth *et al.* 2016; Mavko *et al.* 2020, p. 354). We indicate with the subscripts 1 and 2, the properties of the grains and of the cement, respectively. The model first assumes that the rock is a random pack of grains with porosity close to the critical one ( $\phi_c \approx 0.36$ , a critical porosity) and an average number of grain contacts C = 9. The bulk and shear moduli are

$$K_a = \frac{C}{6}(1-\phi_c)\left(K_2 + \frac{4}{3}\mu_2\right)S_{\perp} \text{ and } \mu_a = \frac{3}{5}\left[K_a + \frac{1}{4}C(1-\phi_c)\mu_2S_t\right],$$
(6)

respectively, where  $K_2$  and  $\mu_2$  are the cement bulk and shear moduli, and  $S_{\perp}$  and  $S_t$  are related to the normal and shear stiffnesses of a cemented two-grain combination, respectively. The explicit expressions of these stiffnesses are given in Mavko *et al.* (2020, p. 355) and depend on the Poisson ratio of the grains  $\nu_1 = (3K_1 - 2\mu_1)/(6K_1 + 2\mu_1)$ , where  $K_1$  and  $\mu_1$  are the grain bulk and shear moduli, and on

$$\alpha = 2 \left[ \frac{\phi_p}{3C(1-\phi_c)} \right]^{1/4},\tag{7}$$

the ratio of the radius of the cement layer to the grain radius, when all cement is deposited at grain contacts. Fig. 1 shows a scheme of the rock after cementation, where  $\phi_p$  is the fraction of cement bonded to the grains, after the water replacement by CO<sub>2</sub> injection.

The rock is an effective medium composed of a binary mixture of cemented grains and loose, unconsolidated ones. Following Avseth *et al.* (2016), the high-porosity limit moduli,  $K_b$  and  $\mu_b$ , are obtained by combining the CCT ( $K_a - \mu_a$ ) and HM ( $K_u - \mu_u$ ) uncemented moduli (see Appendix A) on the basis of the HS upper bound (Appendix B), where the volume fraction of cemented rock to be used in the HS bound is  $\beta_1 = \phi_p/\phi_0$ , i.e., 0 if there is no cement, and 1 if the whole pore-space is filled with cement.

Then, the SC theory is applied to obtain the moduli  $K_s$  and  $\mu_s$  of a medium where all the pores are filled with cement (see Appendix C). There are two phases, grain and cement (subindices 1 and 2). To this purpose, we use eq. (C1) with n = 2,  $\phi_1 = (1 - \phi_0)/(1 - \phi_0 + 1)$ 

 $(\phi_p) = (1 - \phi_0)/(1 - \phi)$ , since  $\phi = \phi_0 - \phi_p$ ,  $\phi_2 = 1 - \phi_1$ ,  $K = K_s$ and  $\mu = \mu_s$ .

Next, we interpolate between the effective high-porosity member given by the HS upper bound and the mineral phase (i.e. zero porosity) using the Voigt–Reuss–Hill average, that is an arithmetic average of the Voigt and Reuss moduli, to obtain the dry-rock bulk and shear moduli:

$$K_m = \frac{1}{2}(K_V + K_W)$$
 and  $\mu_m = \frac{1}{2}(\mu_V + \mu_W)$ , (8)

where

$$K_{V} = (1 - \phi/\phi_{c})K_{s} + (\phi/\phi_{c})K_{b}, \quad \frac{1}{K_{W}} = \frac{1 - \phi/\phi_{c}}{K_{s}} + \frac{\phi/\phi_{c}}{K_{b}},$$
(9)

$$\mu_{V} = (1 - \phi/\phi_{c})\mu_{s} + (\phi/\phi_{c})\mu_{b} \text{ and } \frac{1}{\mu_{W}} = \frac{1 - \phi/\phi_{c}}{\mu_{s}} + \frac{\phi/\phi_{c}}{\mu_{b}}.$$
(10)

Finally, the wet-rock moduli are obtained with the Gassmann equations (e.g. Mavko *et al.* 2020; Carcione 2022). The bulk and shear moduli are given by

$$K_G = K_m + \alpha^2 M, \quad \mu = \mu_m, \tag{11}$$

where

$$M = \left(\frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f}\right)^{-1},\tag{12}$$

$$\alpha = 1 - \frac{\kappa_m}{K_s}.$$
 (13)

and  $K_f$  is the fluid modulus.

#### **3 SEISMIC VELOCITIES**

The P-wave modulus and velocity are

$$E = K_G + \frac{4}{3}\mu_m. \tag{14}$$

and

$$V_P = \sqrt{\frac{E}{\rho}},\tag{15}$$

respectively, where  $\rho$  is the composite density, given by

$$\rho = (1 - \phi) \sum_{i=1}^{2} \beta_i \rho_i + \phi \rho_f,$$
(16)

where  $\rho_i$  and  $\rho_f$  are the densities of the *i*-th solid phase and fluid, respectively.

Similarly, the S-wave velocity is

$$V_S = \sqrt{\frac{\mu}{\rho}},\tag{17}$$

In the initial water-replacement period (no cement), the properties of the water-CO<sub>2</sub> mixture are obtained as

$$K_f = \left(\frac{S_{\rm w}}{K_{\rm w}} + \frac{S_{\rm g}}{K_{\rm g}}\right)^{-1} \tag{18}$$

and

$$\rho_f = S_{\rm w} \rho_{\rm w} + (1 - S_{\rm w}) \rho_{\rm g}, \tag{19}$$

where the subindices 'w' and 'g' refer to water and  $\mathrm{CO}_2$ , respectively.

At high frequencies the pore-fluid pressures are unrelaxed and ultrasonic moduli have to be considered. Adam & Otheim (2013), for instance, use squirt-flow models.

In summary, a workflow of the modelling process is as follows:

(i) Consider the rock as a random pack of grains with porosity close to the critical one and compute the bulk and shear moduli.

(ii) Obtain the high-porosity limit moduli by combining the previous moduli with the HM uncemented moduli on the basis of the Hashin–Shtrikman upper bound.

(iii) Use a SC theory to obtain the moduli of a medium where all the pores are filled with cement (mineral phase).

(iv) Interpolate between the effective high-porosity member and the mineral phase (i.e., zero porosity) using the Voigt–Reuss–Hill average to obtain the dry-rock moduli.

(v) Finally, the wet-rock moduli are obtained with the Gassmann equations.

#### 4 EXAMPLE

#### 4.1 Mineralization and cementation

Let us consider a ferro-basalt rock as in Adam *et al.* (2013), with an iron oxide (Fe<sub>2</sub>O<sub>3</sub>) content greater than 16 per cent. The sample has two pore-size distributions at 0.7–8 and 15–100  $\mu$ m. This difference indicates that there is significant variation in the pore surface area, an important parameter controlling mineralization. A precipitate is developing and after 30 weeks of reactions and carbonate precipitation, porosity decreased from 18.08 per cent to 17.28 per cent. Helium permeability and porosity measurements were performed at a confining pressure of 17.2 MPa. The experiment sets a CO<sub>2</sub>-water mixture at 3.5 mol 1<sup>-1</sup> concentration for the 30 weeks. Pressure and temperature in the vessel are 8.3 MPa and 100 °C, respectively, representative of conditions at 1 km depth, where the gas is supercritical, with low viscosity and a density of a liquid.

In the example we consider three elements, namely, FeCO<sub>3</sub>, CaCO<sub>3</sub> and MgCO<sub>3</sub>, with molar masses of 116, 100 and 84 g mol<sup>-1</sup>, respectively, and with an average value of M = 100 g mol<sup>-1</sup>, used in eq. (3). On the other hand, the specific surface area in Kanakiya *et al.* (2017) of a basalt with 17.6 per cent porosity is 1610 m<sup>-1</sup> (see their table 2), and we set the grain diameter D = 2.6 mm to honour this value. Moreover, we consider the following properties:  $\phi_0 = 18.08$  per cent,  $a = 2.22 \times 10^{-3}$  mol m<sup>-5</sup>,  $b = 10^{-7}$  s<sup>-1</sup>,  $\rho_1 = 2.8$  g cm<sup>-3</sup>, F = 0.7 and V = 1 m<sup>3</sup>. The time step to solve eq. (1) is dt = 8.41 h.

Fig. 2 shows the cement fraction and porosity (a) and the surface area (b) as a function of time. As can be seen, the reduction in porosity follows that of the experiments after 30 weeks of reactions and the surface area is reduced, as expected. From eq. (2) we see that the exponent in eq. (1) is proportional to F/D so that the effect of increasing the grain diameter is similar to that of decreasing the detrital content, i.e., the amount of precipitated cement is less in coarse-grain basalts, because of the reduced surface area, compared to fine-grain basalts.

#### 4.2 Wave velocities

#### 4.2.1 Water replacement

At the initial stage, water is being replaced by  $CO_2$ . We consider that complete replacement takes place on 1 week and that water



**Figure 2.** Cement fraction  $\phi_p$  and porosity  $\phi$  (a), and surface area (b), as a function of time.

saturation  $S_w$  is proportional to the injection time t as

 $S_w = (1 - t [\text{week}])^2.$ 

The grain bulk and shear moduli are  $K_1 = 80.1$  GPa (Adam & Otheim 2013) and  $\mu_1 = 31$  GPa, and we assume  $\rho_1 = 2.8$  g cm<sup>-3</sup>. The conditions are such that the confining and pore pressures are 17.2 and 8.3 MPa, respectively, such that CO<sub>2</sub> is in a supercritial state. At the fluid pressure and temperature of these experiments, the fluid moduli are 0.159 and 2.237 GPa for CO<sub>2</sub> and water, respectively, whereas the respective densities are 832 and 1040 kg m<sup>-3</sup> (Adam & Otheim 2013).

To estimate the velocities, the fluid-mixture bulk modulus is averaged using the Reuss equation and the fluid density is averaged using an arithmetic mean, as in eqs (18) and (19), respectively.

#### 4.2.2 Cementation

During the 30 weeks of injection, elastic waveforms are recorded at 0.5 MHz on dry samples to analyse the changes to the rock frame after basalt and  $CO_2$  have reacted. The samples are taken out of the vessel after 15 weeks and oven dried at 50 °C before recording ultrasonic waveforms again. These steps are then repeated for another 15 weeks (Adam *et al.* 2013). Since the dry-rock velocities are frequency independent, their values can be used at seismic frequencies.

The elastic properties (bulk modulus, shear modulus and density) of the cements are taken from https://materialsproject.org/materia ls/mp-18969/ to be:  $CaCO_3 = 76$ , 32 and 2610;  $FeCO_3 = 111$ , 23



Figure 3. Bulk density as a function of time (a). Panel (b) shows the density in the first 2 weeks.

and 3780; MgCO<sub>3</sub> = 105, 63 and 2900 (in GPa, GPa and kg m<sup>-3</sup>, respectively). Assuming equal composition, and using the Voigt–Reuss–Hill average for the moduli and the arithmetic mean for the density, we obtain the effective elastic properties of the cement, i.e.,  $K_2 = 96$  GPa,  $\mu_2 = 36$  GPa and  $\rho_2 = 3097$  kg m<sup>-3</sup>. The density and bulk modulus of CO<sub>2</sub> are 832 kg m<sup>-3</sup> and 0.159 GPa, respectively (Adam & Otheim 2013). Moreover,  $\delta = 0.01$  MPa and  $p_e = 0$  in the HM model.

Fig. 3(a) displays the bulk density as a function of time. Fig. 3(b) clearly shows the two stages, i.e., water replacement during  $CO_2$  injection, where the density decreases and mineralization and cementation after week 1, where the density starts to increase due to the addition of minerals in the grain contacts (see Fig. 1).

The velocities as a function of time are displayed in Fig. 4. The wet-rock velocity is higher that the dry one for high water saturation, but the opposite trend occurs when water is replaced by  $CO_2$ . The S-wave wet-rock velocity is always lower than the dry one, because the bulk density increases and the shear modulus does not depend on the properties of the fluids. Panel (b) shows the wet-rock P-wave velocity corresponding to the two stages:  $CO_2$  injection, where the saturation of water is zero at 1 week; and mineralization and cementation, where the velocity increases due to grain cementation. Fig. 4(b) explains the behaviour of the velocity shown in the schematic diagram of fig. 4 in Khatiwada *et al.* (2012).



Figure 4. Dry- and wet-rock velocities as a function of time. Panel (b) shows the wet-rock P-wave velocity corresponding to the two stages:  $CO_2$  injection, where the saturation of water is zero at 1 week; and mineralization and cementation, where the velocity increases due to grain cementation.

Varying the cement stiffness one order of magnitude results in only 15 per cent increase in velocities, an effect much smaller than that of increasing cement content [fig. 11 in Dvorkin *et al.* (1994)].

The effect of the grain diameter D on the cement proportion and P-wave velocity is shown in Fig. 5, where the solid and dashed lines correspond to D = 2.6 and 1.3 mm, respectively. In the second case, the surface area is twice that of the first case, and at 30 weeks more cement has been generated and the velocity has slightly increased. In week 10, the velocity difference is only a few metres per second, but as the reaction time increases, the difference increases (in week 30, the difference is about 40 m s<sup>-1</sup>), indicating that it can be resolved by seismic tomography, for example.

In the case presented here, the reaction time is short, but in geological time the variations in P-wave velocity and bulk density will be more pronounced, that is these quantities increase with carbonation whereas porosity decreases, to the extent that these variations can be resolved by seismic methods. The present model can also be applied to other reactive rocks, such as mafic or ultramafic lithologies, like diabase, gabbro and norite (Snaebjörnsdóttir *et al.* 2020).

The CCT model can be further improved by considering the effects of the contact thickness, i.e., is the minimal distance between two adjacent grains, on the normal and tangential stiffness (Guo &



Figure 5. Effect of the grain diameter D on the cement proportion (a) and P-wave velocity (b). The solid and dashed lines correspond to D = 2.6 and 1.3 mm, respectively.

Han 2016), and pre-compaction (Guo & Chen 2022). These effects will be investigated in a future work.

#### **5** CONCLUSIONS

We have developed a quantitative model to simulate the changes in the *P*- and S-wave velocities due to  $CO_2$  injection in basalt, caused by water replacement and mineralization in the form of carbonate cements. The pore space is assumed to be initially saturated with water. The mineralization is based on a model that estimates the volume of precipitated cement and resulting porosity loss, and the velocities as a function of injection time are calculated with a patchy cement model.

In the specific example, the wet-rock velocity is higher that the dry one for high water saturation, but the opposite trend occurs when water is replaced by  $CO_2$ . The S-wave wet-rock velocity is always lower than the dry one, because the bulk density increases and the shear modulus does not depend on the properties of the fluids. The wet-rock P-wave velocity has two opposite trends at the two stages: (i)  $CO_2$  injection, where the saturation of water is zero at week 1; bulk modulus and density decrease as well as velocity and (ii) mineralization and cementation, where the rock is fully saturated with supercritical  $CO_2$ ; density and velocity increases due to grain cementation.

#### DATA AVAILABILITY

No new data were generated or analysed in support of this research. The code underlying this article will be shared on reasonable request to the corresponding author.

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## APPENDIX A: THE HERTZ-MINDLIN MODEL

The Hertz–Mindlin (HM) contact theory, considers a random packing of spheres as an effective medium (Hertz 1895; Mindlin 1949; Walton 1987; Mavko *et al.* 2020). We modify the HM model by replacing the effective pressure  $p_e$ , by the augmented value,  $p_e + \delta$ , following Gangi & Carlson (1996), assuming that the grains are subjected to an initial level of bonding determined by  $\delta$  (a pre-stress condition). A HM model with augmented effective pressure is given in Carcione *et al.* (2007) to model bonded grains at  $p_e = 0$ .

Then, the bulk and shear (uncemented) moduli at the critical porosity  $\phi_c$  are given by

$$K_u = \left[\frac{C^2(1-\phi_c)^2 \mu_1^2(p_e+\delta)}{18\pi^2(1-\nu_1)^2}\right]^{1/3}$$
(A1)

and

$$\mu_u = \frac{3(5 - 4\nu_1)}{5(2 - \nu_1)} K_u,\tag{A2}$$

where  $\mu_1$  is the shear modulus of the grains,  $\nu_1$  is the Poisson ratio of the grains and *C* is the average number of contacts per spherical grain.

#### APPENDIX B: HASHIN AND SHTRIKMAN BOUNDS AND AVERAGES

Let us denote the solid bulk and shear moduli by  $K_i$  and  $\mu_i$ , respectively. A two-solid composite, with no restriction on the shape of the two phases, has stiffness bounds given by the Hashin & Shtrikman (1963) equations,

$$K_{\rm HS}^{\pm} = K_1 + \frac{\beta_2}{(K_2 - K_1)^{-1} + \beta_1 \left(K_1 + \frac{4}{3}\mu_\beta\right)^{-1}}$$
(B1)

and

$$\mu_{\rm HS}^{\pm} = \mu_1 + \frac{\beta_2}{(\mu_2 - \mu_1)^{-1} + \beta_1 \left[\mu_1 + \frac{\mu_\beta}{6} \left(\frac{9K_\beta + 8\mu_\beta}{K_\beta + 2\mu_\beta}\right)\right]^{-1}},$$
(B2)

where  $\beta_1$  and  $\beta_2$  are the fractions of solid 1 and 2 ( $\beta_1 + \beta_2 = 1$ ), and  $\beta_1 = \phi_1/(1 - \phi)$ . We obtain the upper bounds when  $K_\beta$  and  $\mu_\beta$ are the maximum bulk and shear moduli of the single components, and the lower bounds when these quantities are the corresponding minimum moduli, that is we have the upper bound if 1 is the stiffer medium and the lower bound is obtained if 1 is the softer medium (Mavko *et al.* 2020).

The arithmetic averages of the bounds are frequently used to obtain the bulk and shear moduli of a mineral mixture, i.e.,

$$K_s = \frac{1}{2}(K_{\rm HS}^+ + K_{\rm HS}^-), \quad \mu_s = \frac{1}{2}(\mu_{\rm HS}^+ + \mu_{\rm HS}^-).$$
 (B3)

#### APPENDIX C: SELF-CONSISTENT (SC) SCHEME

In the SC approximation, the elastic moduli of an unknown effective medium have to be found implicitly. The model has been used by Gurevich & Carcione (2000) to obtain the stiffnesses of sand–clay mixtures, where the inclusions are spherical. Here, we consider spherical grains (aspect ratio  $\gamma = 1$ ) and pores of aspect ratio  $\gamma < 1$ . In this case, it is N = 3 for grain, cement and water.

The effective bulk and shear moduli of the composite medium (*K* and  $\mu$ ), with *N* phases and proportion  $\phi_i$ , are obtained as the roots of the following system of equations

$$\sum_{i=1}^{N} \phi_i (K_i - K) P_i = 0,$$
  
$$\sum_{i=1}^{N} \phi_i (\mu_i - \mu) Q_i = 0,$$
 (C1)

where

$$P_{i} = \frac{K + \frac{4}{3}\mu}{K_{i} + \frac{4}{3}\mu}, \quad i = 1, \dots, n$$

$$Q_{i} = \frac{\mu + \zeta}{\mu_{i} + \zeta},$$
(C2)

$$\zeta = \frac{\mu}{6} \cdot \frac{9K + 8\mu}{K + 2\mu}$$

for the grains, i.e., i = 1, 2 (Mavko *et al.* 2020) and  $P_3 = \frac{1}{3}T_{iijj}$  and  $Q_3 = \frac{1}{5}(T_{ijij} - P)$  (for the pores), where  $T_{iijj}$  and  $T_{ijij}$  are given in appendix A of Berryman (1980) or in p. 246 of Mavko *et al.* (2020). If  $\gamma = 1, P_3$  and  $Q_3$  are given by eq. (C2). A limitation of this theory is that the inclusions are isolated, so that pore pressures are not equilibrated and the model computes high-frequency velocities.

To solve eq. (C1), we use the algorithm developed by Goffe *et al.* (1994). The Fortran code can be found in: https://econwpa.ub.uni -muenchen.de/econ-wp/prog/papers/9406/9406001.txt.