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Gassmann Modeling of Acoustic Properties of Sand-clay Mixtures

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Abstract—The feasibility of modeling elastic properties of a fluid-saturated sand-clay mixture rock is analyzed by assuming that the rock is composed of macroscopic regions of sand and clay. The elastic properties of such a composite rock are computed using two alternative schemes.

The first scheme, which we call the composite Gassmann (CG) scheme, uses Gassmann equations to compute elastic moduli of the saturated sand and clay from their respective dry moduli. The effective elastic moduli of the fluid-saturated composite rock are then computed by applying one of the mixing laws commonly used to estimate elastic properties of composite materials.

In the second scheme which we call the Berryman-Milton scheme, the elastic moduli of the dry composite rock matrix are computed from the moduli of dry sand and clay matrices using the same composite mixing law used in the first scheme. Next, the saturated composite rock moduli are computed using the equations of Brown and Korringa, which, together with the expressions for the coefficients derived by Berryman and Milton, provide an extension of Gassmann equations to rocks with a heterogeneous solid matrix.

For both schemes, the moduli of the dry homogeneous sand and clay matrices are assumed to obey the Krief's velocity-porosity relationship. As a mixing law we use the self-consistent coherent potential approximation proposed by Berryman.

The calculated dependence of compressional and shear velocities on porosity and clay content for a given set of parameters using the two schemes depends on the distribution of total porosity between the sand and clay regions. If the distribution of total porosity between sand and clay is relatively uniform, the predictions of the two schemes in the porosity range up to 0.3 are very similar to each other. For higher porosities and medium-to-large clay content the elastic moduli predicted by CG scheme are significantly higher than those predicted by the BM scheme.

This difference is explained by the fact that the BM model predicts the fully relaxed moduli, wherein the fluid can move freely between sand and clay regions. In contrast, the CG scheme predicts the no-flow or unrelaxed moduli. Our analysis reveals that due to the extremely low permeability of clays, at seismic and higher frequencies the fluid has no time to move between sand and clay regions. Thus, the CG scheme is more appropriate for clay-rich rocks.

Key words: Wave velocities, porous medium, clay content, poroelasticity.

1. *Introduction*

Gassmann formulas relate the elastic moduli of a fluid-saturated porous material to those of the dry (empty) matrix and fluid compressibility. They are widely

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used in modeling acoustic properties of fluid-saturated rocks, notably sands, sandstones, and limestones. Application of the same equations to clay-sand mixtures, such as shaly sandstones and shales, is not straightforward, because Gassmann formulas are not valid, in a strict sense, for materials with a heterogeneous solid matrix (BROWN and KORRINGA, 1975; BERRYMAN and MILTON, 1991). Thus the use of the Gassmann equation in models of sand-clay mixtures (XU and WHITE, 1995; GOLDBERG and GUREVICH, 1998) should be regarded as an approximation, which may be accurate enough only for some geometrical distributions of sand and clay particles within the solid matrix. One such configuration is when sand and clay particles are mixed very "homogeneously," so that the solid matrix can be considered as an aggregate of composite grains, each grain being a mixture of sand and clay particles. We refer to a model based on this assumption, such as the one discussed by GOLDBERG and GUREVICH (1998), as a homogenized matrix model or HM.

In the present paper we consider a different situation, in which the rock consists of regions of sand and clay, which are much larger than the characteristic pore or grain size (Fig. 1). The Gassmann formulas for the whole material clearly do not apply in this case, and alternative approaches must be employed. Two possible models are investigated below. One approach is to calculate the properties of the dry-sand and "dry-clay" matrices, use Gassmann equations to obtain the moduli of the saturated sand and clay, and then apply a certain mixing law to obtain the moduli of the fluid-saturated rock as a composite material consisting of two

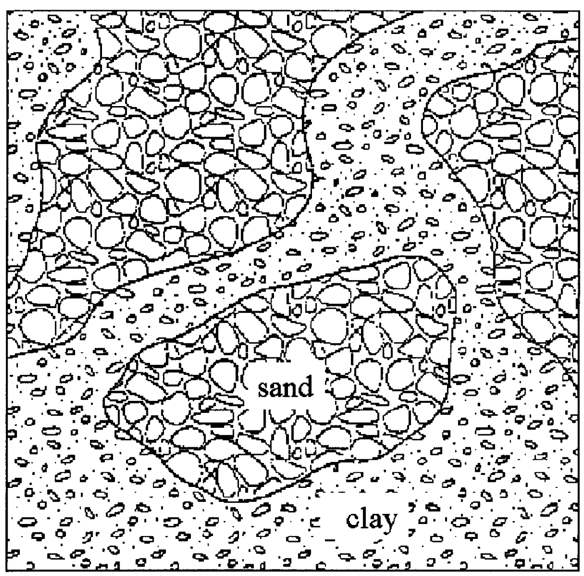


Figure 1
The rock is composed of macroscopic sand and clay regions.

constituents, fluid-saturated sand and fluid-saturated clay. We call this approach a composite Gassmann model (CG).

Another alternative is based on the equations of BROWN and KORRINGA (1975), who generalized GASSMANN (1951) formulas to materials with heterogeneous solid matrix. The equations of Brown and Korringa contain four bulk elastic parameters which characterize the solid matrix, compared with two parameters (bulk modulus of the solid grains and that of the dry matrix) for the Gassmann equations. The additional two parameters of Brown and Korringa are, in general, difficult to estimate. However, BERRYMAN and MILTON (1991) demonstrated that, when the matrix consists of macroscopic homogeneous regions which are large enough to be characterized by effective elastic constants, the two additional elastic constants of Brown and Korringa can be related to the moduli of the two constituent dry matrices and the moduli of the composite dry matrix. In other words, if we know the properties of the two dry matrices, we can apply a composite mixing law to compute the moduli of the composite dry matrix, then compute Brown-Korringa parameters using the equations of BERRYMAN and MILTON (1991), and, finally, calculate the properties of the fluid-saturated rock using the equations of BROWN and KORRINGA (1975). This model is called Berryman-Milton model or BM.

In this paper we analyze the feasibility of using the CG and BM models to calculate elastic properties of sand-clay mixtures. The paper is organized as follows. First, we define the geometrical model of a sand-clay mixture under investigation. Then, we describe the CG and BM models and their implementation. Finally, we apply both procedures to the same material, compute the corresponding velocity-porosity relationships, and compare the predictions of the CG and BM models with each other and with the homogenized matrix model (HM) of GOLDBERG and GUREVICH (1998).

2. Geometrical Model

We consider our rock to be a mixture of three materials: non-clay mineral (e.g., quartz), clay particles, and a fluid. These constituents are characterized by densities ρ_{sand}, ρ_{clay}, and ρ_f, bulk moduli K^m_{sand}, K^m_{clay}, and K_f, and shear moduli μ^m_{sand}, μ^m_{clay}, and 0. The volume fraction of clay in the solid portion of the rock is C and the total porosity is ϕ. The geometrical distribution of clay and quartz grains is such that the rock consists of macroscopic regions where the solid component is either pure quartz or pure clay. "Macroscopic" here means that the size of these regions is substantially larger than the characteristic grain size, so that they may be characterized by porosities ϕ_{sand}, ϕ_{clay} and effective elastic constants K^0_{sand}, K^0_{clay} and μ^0_{sand}, μ^0_{clay}, respectively. The volume fractions of these sand and clay regions are denoted by f_{sand} and f_{clay}, so that

$$f_{\text{sand}} + f_{\text{clay}} = 1.$$

These volume fractions can be uniquely related to the clay content C and the sand and clay porosities ϕ_{sand} and ϕ_{clay}. Indeed, if the volume of all solid particles in a unit volume of the rock is $1 - \phi$, then the volume of solid clay particles V_{clay} in the same unit volume of the rock is $C(1 - \phi)$. On the other hand, V_{clay} can be considered as a solid part of the clay matrix, i.e., $V_{\text{clay}} = (1 - \phi_{\text{clay}})f_{\text{clay}}$. Therefore,

$$C(1 - \phi) = (1 - \phi_{\text{clay}})f_{\text{clay}}$$

or

$$f_{\text{clay}} = \frac{C(1 - \phi)}{1 - \phi_{\text{clay}}}. \tag{1}$$

Similarly,

$$f_{\text{sand}} = \frac{(1 - C)(1 - \phi)}{1 - \phi_{\text{sand}}}. \tag{2}$$

Our aim is to determine the effective elastic moduli (and compressional and shear velocities) of the composite rock as a function of clay content C and porosity ϕ.

To do this, we must specify the values of ϕ_{sand} and ϕ_{clay}, which define the distribution of the total porosity

$$\phi = \phi_{\text{sand}}f_{\text{sand}} + \phi_{\text{clay}}f_{\text{clay}} \tag{3}$$

between the two porous matrices, and are usually unknown. By definition, $\phi_{\text{sand}} = \phi$ for pure sand ($C = 0$), and $\phi_{\text{sand}} = 0$ for pure shale ($C = 1$). We thus assume that for a rock with the clay content C, the sand porosity is given by the equation

$$\phi_{\text{sand}} = \phi(1 - C)^{\gamma} \tag{4}$$

where $\gamma \geq 0$ is a parameter of porosity distribution, a dimensionless number. The corresponding "clay" porosity can be found by substituting expressions (1) and (2) into equation (3):

$$\frac{1 - C}{1 - \phi_{\text{sand}}} + \frac{C}{1 - \phi_{\text{clay}}} = \frac{1}{1 - \phi}. \tag{5}$$

Solving equation (5) for ϕ_{clay} yields

$$\phi_{\text{clay}} = \frac{\phi - \phi_{\text{sand}}(1 - C + \phi C)}{C + \phi - C\phi - \phi_{\text{sand}}}. \tag{6}$$

Equation (4) implies that for a clean sand all the pores are within the sand matrix, $\phi_{\text{sand}} = \phi$, whereas for high clay content and $\gamma \neq 0$ the isolated sand grains are surrounded by a clay-fluid mixture containing all the porosity. For $\gamma = 0$ we have

$$\phi_{\text{sand}} = \phi_{\text{clay}} = \phi,$$

so that the porosities are the same for both sand and clay matrices, whatever the clay content. On the other hand, for a clay-bearing rock setting $\gamma = \infty$ implies that the solid grains contain no pores but are surrounded by a clay-fluid mixture, meaning

$$\phi_{\text{sand}} = 0, \tag{7}$$

$$\phi_{\text{clay}} = \frac{\phi}{\phi + C(1-\phi)}. \tag{8}$$

3. Computational Schemes

Elastic Moduli for a Homogeneous Dry Matrix

In both the CG and BM models we need to define the bulk and shear moduli of the macroscopic constituents K^0_{sand}, K^0_{clay} and μ^0_{sand}, μ^0_{clay} as functions of the corresponding porosities ϕ_{sand} and ϕ_{clay}. Following GOLDBERG and GUREVICH (1998), we employ here a modified Krief model (KRIEF *et al.*, 1990),

$$\sigma_i = 1 - (1-\phi_i)^{A_i/(1-\phi_i)}, \tag{9}$$

where $\sigma_i = 1 - K^0_i/K^m_i$, i is either *sand* or *clay*, and A_i is a dimensionless number which defines the steepness of the velocity-porosity curve (for an idealized material with spheroidal pores A_i can be approximately related to the dominant pore aspect ratio, see XU and WHITE, 1995). Any other known velocity-porosity relationship (see e.g., MAVKO *et al.*, 1998) can be used instead.

Assume that the dry matrix moduli K^0_{sand}, K^0_{clay} and μ^0_{sand}, μ^0_{clay} are known. The moduli of the corresponding saturated homogeneous constituents can be obtained from Gassmann equations,

$$K^{\text{sat}}_i = K^0_i + \sigma^2_i M_i$$

where M_i is so-called pore space modulus given by

$$\frac{1}{M_i} = \frac{\sigma_i}{K^m_i} + \phi\left(\frac{1}{K_f} - \frac{1}{K^m_i}\right)$$

and i is either *sand* or *clay*. The shear moduli are not affected by the saturation, so that

$$\mu^{\text{sat}}_i = \mu^0_i.$$

Once the saturated moduli are known, the bulk and shear moduli of the saturated composite rock may be computed by applying one of the mixing laws used to compute the elastic properties of composite materials (CHRISTENSEN, 1979; BERRYMAN, 1995; MAVKO *et al.*, 1998). We consider the choice of the mixing law in a later section.

Berryman-Milton Model

Beginning again with the dry matrix moduli K^0_{sand}, K^0_{clay} and μ^0_{sand}, μ^0_{clay}, the bulk and shear moduli of the dry composite matrix K^0_* and μ^0_* may be computed by applying one of the composite mixing laws discussed in the next section. Thereafter, the moduli of the saturated composite rock can be computed using the formulas of Brown and Korringa

$$K^{\text{sat}} = K^0 + \sigma^2 M \tag{10}$$

$$\frac{1}{M} = \frac{\sigma}{K_s} + \phi\left(\frac{1}{K_f} - \frac{1}{K_\phi}\right), \tag{11}$$

$$\sigma = 1 - K^0_*/K_s, \tag{12}$$

and

$$\mu^{\text{sat}} = \mu^0_*, \tag{13}$$

where K_s, and K_ϕ are constants that depend on the moduli of the matrix constituents and their geometrical distribution. For a rock consisting of macroscopic regions, each having a homogeneous matrix, the equations for K_s and K_ϕ have been derived by Berryman and Milton (1991)

$$\frac{\sigma - \sigma_{\text{sand}}}{\sigma_{\text{clay}} - \sigma_{\text{sand}}} = \frac{K^0_* - K^0_{\text{sand}}}{K^0_{\text{clay}} - K^0_{\text{sand}}}, \tag{14}$$

$$\frac{\phi}{K_\phi} = \frac{\sigma}{K_s} - \sum_i \frac{\sigma_i - \phi_i}{K^m_i} f_i + \left(\sum_i \sigma_i f_i - \sigma\right)\left(\frac{\sigma_{\text{sand}} - \sigma_{\text{clay}}}{K^0_{\text{sand}} - K^0_{\text{clay}}}\right), \tag{15}$$

where i refers to either *sand* or *clay*. Once K^0_* and μ^0_* have been determined, we can use (14) to compute σ, then use eq. (12) to evaluate K_s, compute K_ϕ from eq. (15), and, finally, evaluate the saturated bulk modulus using the Brown-Korringa equations (10)–(13).

Composite Mixing Law

Lower Hashin-Shtrikman bound

Both the CG and BM schemes require a composite mixing law relating the elastic moduli of a composite material to the elastic moduli and volume fractions of the constituent materials. This is a classical problem in mechanics of composite materials (Christensen, 1979). The elastic moduli of a composite are not uniquely defined by the moduli and volume fractions of constituents, but depend on the geometrical distribution of the constituents in the composite. However, the range of possible moduli of the composite is not infinite; for an isotropic geometry, its bulk and shear moduli must lie within the so-called Hashin-Shtrikman bounds (Christensen, 1979; Berryman, 1995). The lower (upper) Hashin-Shtrikman bound corresponds to the geometrical distribution of the constituents such that the softer (harder) material serves as the primary load-bearing phase. For any other isotropic geometrical distribution of the constituents the moduli lie between these two bounds.

A clay-sand mixture can be thought of as a material with two constituents, in which sand is the harder and clay is the softer constituent. In many (though clearly not in all) situations the primary load bearing constituent is clay. For shales and sandy shales this is quite obvious. Shaly sandstones are also believed to have a fair amount of clay between the sand grains. For this reason, the lower Hashin-Shtrikman bound is sometimes used as a mixing law for sand-clay mixtures, such as in the homogenized matrix model of GOLDBERG and GUREVICH (1998).

However, for the particular geometry considered in this paper, the use of the lower Hashin-Shtrikman bound as a mixing law is not fully justified. Indeed, it implies that all macroscopic sand regions are completely surrounded by macroscopic clay regions. In particular, if the parameter γ is high ($\gamma \geq 1$), it means that most of the porosity is contained within the clay matrix ($\phi_{sand} \ll \phi$), i.e., all sand grains are surrounded by the clay matrix containing most of the pore space. This configuration may be appropriate for shales, however for sandstones with a moderate clay content ($C < 0.3$) it leads to high values of clay porosity ϕ_{clay}, see equation (8). This would mean that the solid grains are surrounded by a suspension of clay particles in the fluid, which thus becomes the load-bearing phase. This picture is very unrealistic and leads to a steep decrease in both bulk and shear moduli of the rock with porosity (Fig. 2) for modest clay concentrations and to a decidedly less steep decrease for high clay content, thus creating a crossover point at a porosity of about 0.27. The only condition in which the low Hashin-Shtrikman bound yields reasonable values of compressional and shear velocities is when the porosity is distributed uniformly between the sand and clay matrices ($\gamma = 0$, or $\phi_{sand} = \phi_{clay} = \phi$, see Fig. 3). But this does not create a more realistic circumstance.

Self-consistent scheme

In order to choose a more realistic mixing law, we must first think of a realistic geometrical relationship between sand and clay regions. It is reasonable to assume that when the sand component dominates ($C \ll 1$, $f_{sand} \gg f_{clay}$), the sand will be a continuous phase and the clay matrix will form isolated inclusions, and *vice versa*. That is, the dominant component is the load-bearing phase. Such a configuration is modeled in a self-consistent (SC) scheme proposed by BERRYMAN (1980a,b). This scheme is sometimes called the self-consistent coherent potential approximation or CPA and can be considered as an extension of the well-known theory of KUSTER and TOKSÖZ (1974) to arbitrary volume fractions of the constituents (BERRYMAN, 1995). This mixing law has an additional parameter v that denotes the aspect ratio of the inclusions, which are assumed ellipsoidal in shape. In the CPA scheme, the effective bulk K^* and shear μ^* moduli of a composite consisting of two constituents 1 and 2 with volume fractions f_1 and f_2, bulk moduli K_1 and K_2, and shear moduli μ_1 and μ_2, are obtained as the roots of the following system of equations

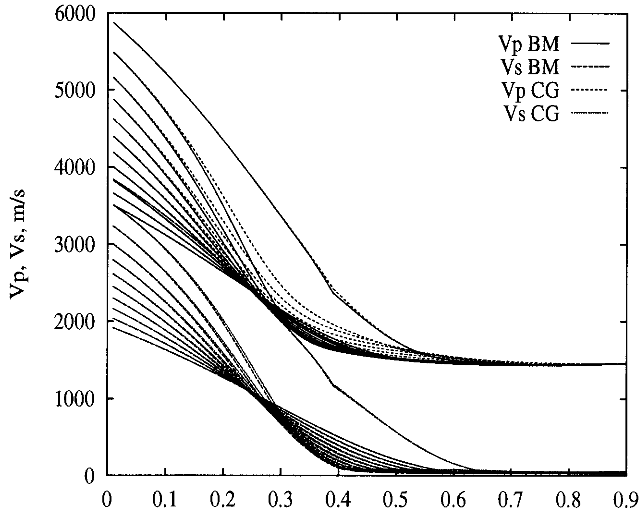


Figure 2
Compressional and shear velocities computed with Berryman-Milton (BM) and Composite Gassmann (CG) schemes versus porosity for different clay content using the lower Hashin-Shtrikman bound as a mixing law. The lines showing lower V_p and V_s at zero porosity correspond to higher clay content. The porosity distribution parameter γ is 1.

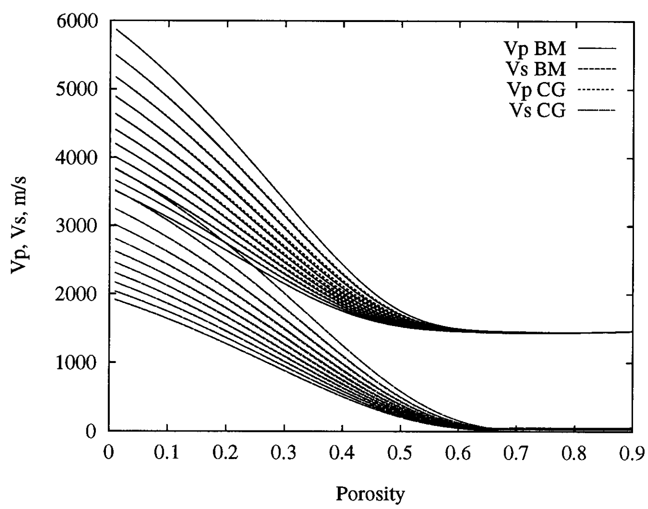


Figure 3
Compressional and shear velocities computed with Berryman-Milton (BM) and Composite Gassmann (CG) schemes versus porosity for different clay content using the lower Hashin-Shtrikman bound as a mixing law. The lines showing lower V_p and V_s at zero porosity correspond to higher clay content. The porosity distribution parameter γ is 0, which corresponds to the homogeneous distribution of porosity between sand and clay, $\phi_{\text{sand}} = \phi_{\text{clay}} = \phi$.

$$\sum_{i=1}^{2} f_i(K_i - K^*)P_i = 0 \tag{16}$$

$$\sum_{i=1}^{2} f_i(\mu_i - \mu^*)Q_i = 0 \tag{17}$$

where P_i and Q_i are given by simple but cumbersome analytical expressions involving K_i, μ_i, K^*, μ^*, and the shape factor v, see BERRYMAN (1980b). For spherical inclusions ($v = 1$) the functions P_i and Q_i are given by

$$P_i = \frac{K^* + \frac{4}{3}\mu^*}{K_i + \frac{4}{3}\mu^*}$$

$$Q_i = \frac{\mu^* + F^*}{\mu_i + F^*}$$

with

$$F^* = \frac{\mu^*}{6}\frac{9K^* + 8\mu^*}{K^* + 2\mu^*}\,.$$

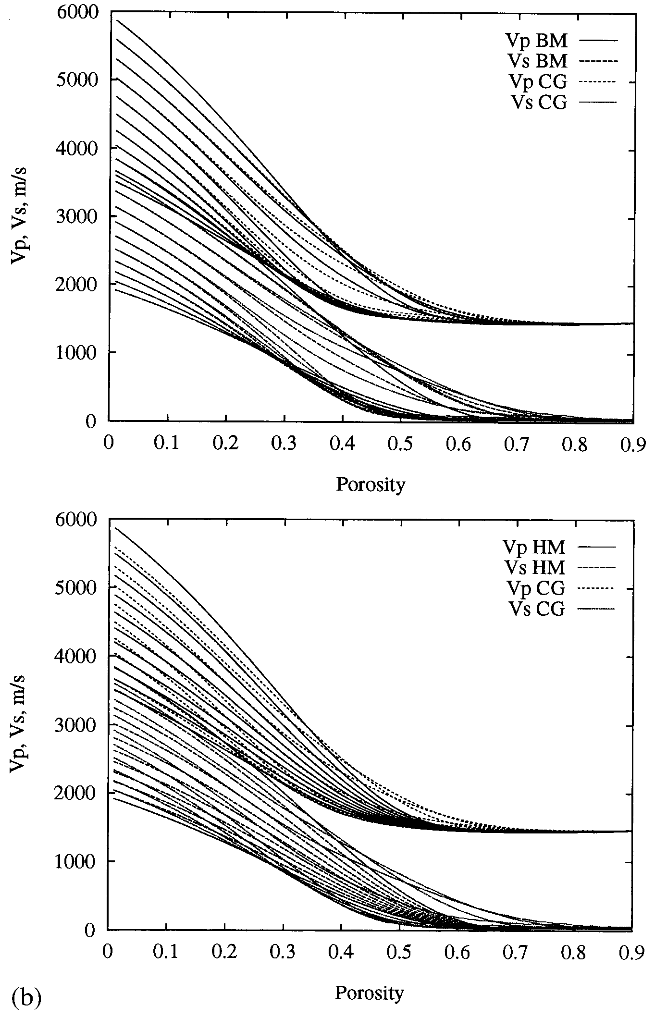
We use the CPA scheme in our numerical examples, solving equations (16)–(17) for K^* and μ^* by iteration.

4. *Numerical Results*

To analyze the predictions of the two schemes we have computed compressional and shear velocities as functions of porosity and clay content for a set of parameters typical for shaly sandstones: $\rho_{\text{sand}} = \rho_{\text{clay}} = 2.65$ g/cm^3, $\rho_f = 1$ g/cm^3, bulk moduli $K^m_{\text{sand}} = 40$ GPa, $K^m_{\text{clay}} = 20$ GPa, $K_f = 2.25$ GPa, shear moduli $\mu^m_{\text{sand}} = 40$ GPa, $\mu^m_{\text{clay}} = 10$ GPa, and Krief's exponents $A_{\text{sand}} = 3.0$, $A_{\text{clay}} = 3.5$. Figures 4a–b show the predictions of the CG and BM schemes compared with the HM model of Goldberg and Gurevich. The porosity distribution parameter γ was 1 and the shape of inclusions in the composite model was assumed spherical ($v = 1$). The figures that follow show the corresponding predictions for $\gamma = 1$, $v = 0.2$ (Fig. 5), $\gamma = 0$, $v = 1$ (Fig. 6), $\gamma = 20$, $v = 1$ (Fig. 7).

From Figures 4–7 we can make the following observations:

- For relatively small values of the porosity distribution parameter γ ($\gamma \leq 1$), the predictions of the CG and BM schemes in the usual reservoir porosity range ($0 < \phi < 0.3$) are quite similar both qualitatively and quantitatively.
- In the same porosity range these predictions have the same general trends as the homogenized matrix (HM) scheme. The visible difference in velocity-porosity slope can be accounted for by a slight change in the Krief's exponents in different schemes. Indeed, Krief's exponent need not be taken the same for different models.



(b)

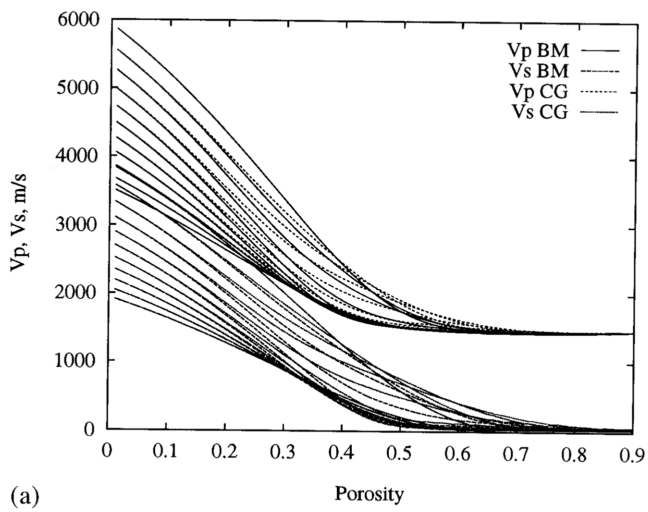
Figure 4

Compressional and shear velocities computed with Berryman-Milton (BM) and Composite Gassmann (CG) schemes versus porosity for different clay content using the Coherent Potential Approximation as a mixing law. The lines delineating lower V_p and V_s at zero porosity correspond to higher clay content. (a) Comparison of BM and CG schemes; (b) Comparison of Homogenized-matrix (HM) and CG schemes. The porosity distribution parameter γ is 1; aspect ratio of inclusion $v = 1$ (spherical inclusions).

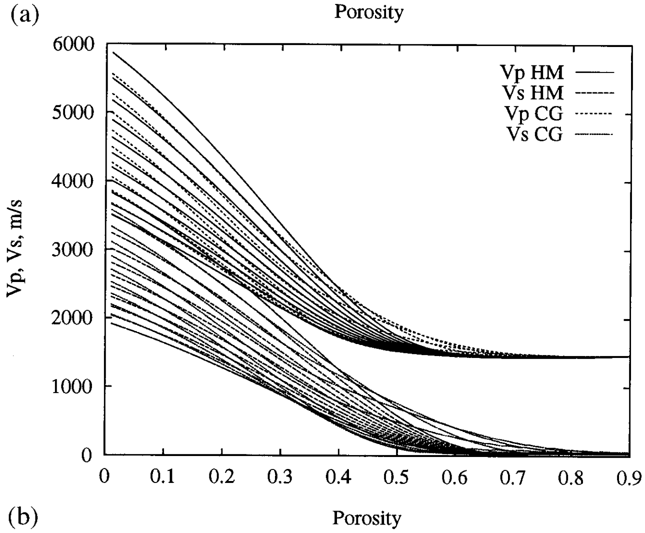
- For larger values of γ ($\gamma \geq 1$) at high porosities, both the CG and BM schemes predict certain crossover points at small and large clay contents. In particular, in the high C range (shale) the curves for lower clay content decrease with porosity more rapidly than those with higher clay content, so that above a certain porosity value all these velocities are approximately identical. This behavior is consistent with the concept of critical porosity (Yin *et al.*, 1994; Mavko *et al.*, 1998).

- For these moderate to large values of γ the predictions of the CG and BM schemes exhibit significant velocity differences at high porosities and medium clay content.
- For uniform distribution of porosity between the sand and clay matrices ($\gamma = 0$), the predictions of the CG, BM, and HM schemes are almost identical.
- Large values of γ ($\gamma > 10$) lead to unrealistically steep decreases of velocities with porosity even in a low porosity range.

These observations are analyzed in the next section.



(a)



(b)

Figure 5

The same as Figures 4a,b but for $\gamma = 1$, $\nu = 0.2$.

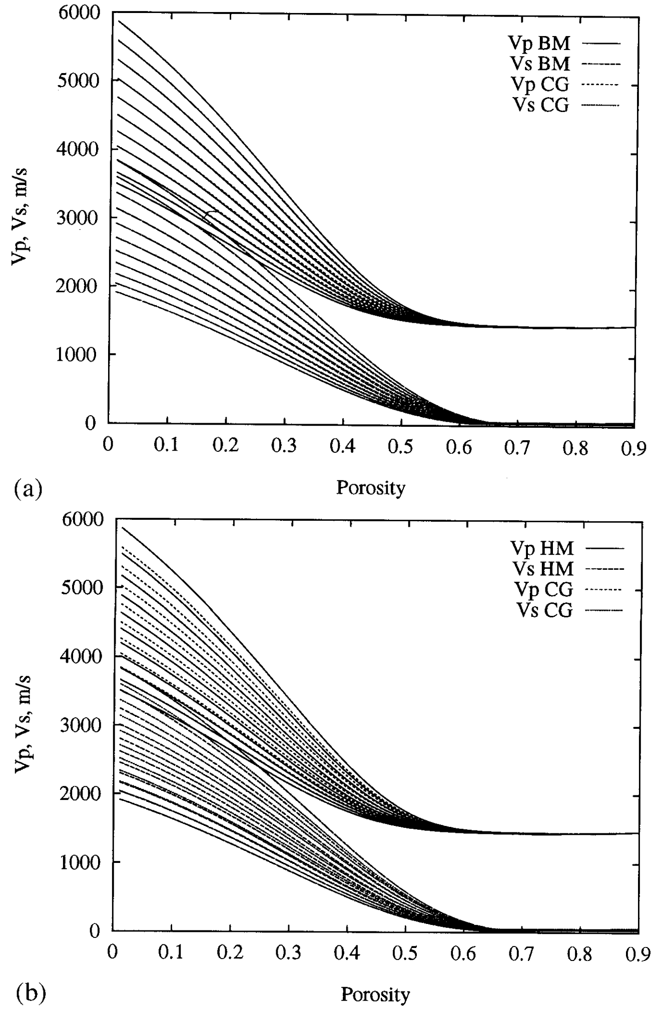


Figure 6
The same as Figures 4a,b but for $\gamma = 0$, $v = 1$.

5. Discussion

In this paper we have used the composite Gassmann (CG) and Berryman-Milton (BM) schemes to model the compressional and shear velocities in sand-clay mixtures as functions of porosity and clay content. By construction, both schemes are thought to be rigorous and exact, meaning that with the correct elastic moduli of the dry sand and clay matrices K^0_{sand}, K^0_{clay} and μ^0_{sand}, μ^0_{clay}, and the correct mixing law, they should predict the moduli of the composite saturated rock exactly. Thus a basic question arises: *why are the predictions of the CG and BM schemes different?*

Indeed, the difference observed cannot be explained by the use of the approximate modulus-porosity law of Krief [equation (9)]. Though the moduli may not be exact for any rock, they have been taken the same for both schemes. Likewise the coherent potential approximation we used as a composite mixing law is approximate, although it is known to be physically realizable (MILTON, 1985), in the sense that there exists a geometrical configuration of the two constituents for which the effective elastic moduli predicted by CPA are exact. Thus the moduli of the composite saturated material predicted by the CG and BM schemes should also be exact. The fact that they differ poses two questions:

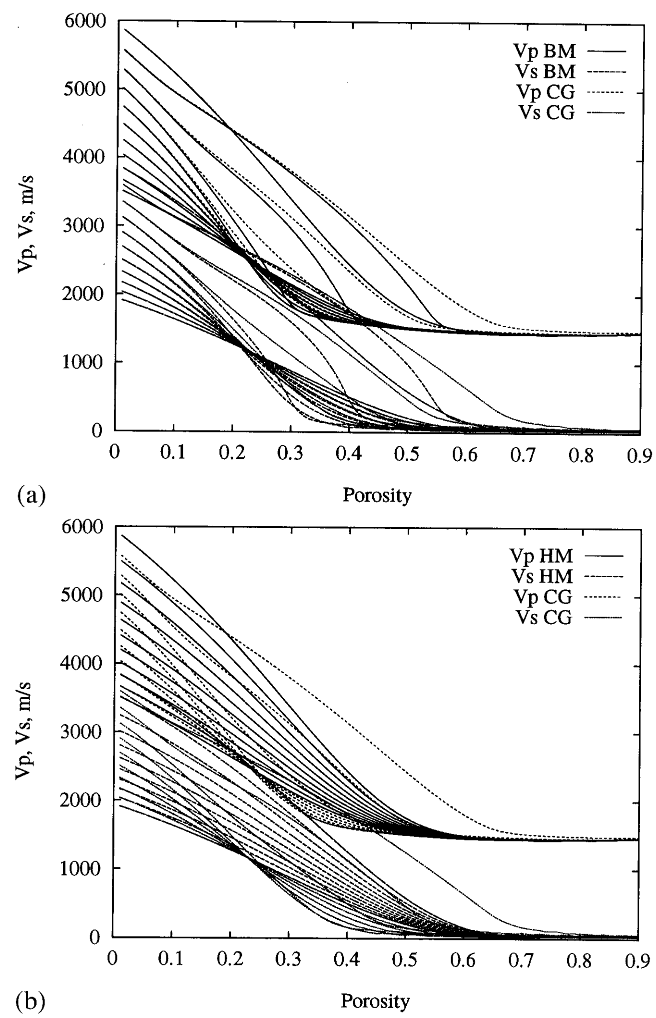


Figure 7
The same as Figures 4a,b but for $\gamma = 10$, $\nu = 1$.

1. What are the physical reasons behind the difference observed?
2. Which of these schemes is more adequate for modeling rock properties in seismics/geoacoustics?

To answer these questions, we must revert to the construction of these schemes. The BM scheme is based on a rigorous mechanical extension of Gassmann equations to account for the heterogeneity of the solid matrix. On the other hand, the CG scheme applies Gassmann equations to compute the saturated moduli of the sand and clay, and then utilizes these moduli to predict the properties of the composite rock. Since the composite mixing law was designed for elastic composites, we effectively replace both saturated sand and clay with the equivalent elastic materials. This may sound slightly arbitrary, since both materials are poroelastic rather than elastic. In particular, by doing so, we neglect a possible flow of the pore fluid between the sand and clay matrices, effectively ignoring one of the degrees of freedom in the system. For matrices that differ substantially in their compliances, the compression of the composite rock may lead to significant movement of the pore fluid from more compliant to less compliant regions, thus reducing the overall stiffness. Ignoring this effect by effectively sealing the boundaries between sand and clay regions may lead to an overestimation of the effective elastic moduli.

We could just conclude from this analysis that the BM scheme is rigorous, while the CG is a no-flow approximation that may or may not be accurate enough in a given situation. However, a more thoughtful look into the above analysis suggests that this conclusion is not as obvious as it may appear. As mentioned above, the BM scheme implicitly takes into account a possible fluid flow between the two constituents under the deformation. This implies that the deformation is slow enough for the fluid to flow from one constituent to the other (the so-called relaxed conditions). To analyze whether this is the case for a given situation, we must make our analysis dynamic rather than static, i.e., consider the frequency. Indeed, it has been shown (GUREVICH and LOPATNIKOV, 1995; GUREVICH *et al.*, 1998) that the fluid can be considered as fully relaxed below the characteristic frequency

$$\omega_0 \sim \frac{\kappa K_f}{\phi \eta h^2}$$

where η is the fluid viscosity, h is the characteristic size of the constituent regions, and κ is the permeability that characterizes the flow between the constituents. In general κ is average permeability, although for rocks composed of constituents whose permeabilities differ by orders of magnitude, it is obviously controlled by the lowest permeability. In the case of sand-clay mixtures, κ is dominated by the permeability of clay or shale. For frequencies $\omega \gg \omega_0$, the fluid has no time to move from one constituent matrix to another, therefore the rock is characterized by no-flow moduli, which can be obtained by the CG scheme.

To obtain an upper limit for the characteristic frequency ω_0 we recall that h is the characteristic size of the constituent regions. For these regions to be macroscopic h must be large compared with the main grain size. For sand-clay mixtures, the largest possible grain size is 1 mm. We thus can take 1 cm as the lower estimate for h. Assuming water saturation ($K_f = 2.25$ GPa, $\eta = 10^{-3}$ Pa s), porosity of 0.2, and maximum clay permeability of $\kappa = 10^{-4}$ Darcy or 10^{-16} m^2, we obtain $\omega_0 \leq 10$ s^{-1}, which approximately corresponds to a circular frequency of 1.6 Hz. In fact, for most shales the permeability is much lower than 10^{-16} m^2 (BEST and KATSUBE, 1995). This means that the relaxed moduli as predicted by the BM model are only relevant for frequencies below 1 Hz. For seismic and higher frequencies, the unrelaxed moduli as predicted by the CG scheme should be used. We should emphasize that this conclusion is a direct result of the extremely low permeability of shales, and thus is only relevant for sand-clay mixtures. For other heterogeneous porous rocks ω_0 can be much higher, in which case the relaxed moduli as predicted by the Berryman-Milton scheme may be relevant.

We have explained and analyzed the difference between the compressional and shear velocities in clay-sand mixtures predicted by CG and BM schemes. The physical significance of the other observations is not as obvious. In particular, the crossover at large clay content and large-to-medium porosity may be characteristic of the particular distribution of porosity between the sand and clay regions, as defined by equations (4) and (6), rather than of the models themselves. More studies of the microstructure of real rocks in the wide range of porosity and clay content are needed to define a more realistic distribution of porosity between sand and clay regions.

6. *Conclusions*

We have compared two schemes for modeling the elastic properties of a rock composed of macroscopic sand and clay regions. The composite Gassmann (CG) scheme uses Gassmann equations to compute elastic moduli of the saturated sand and clay from their respective dry moduli. The effective elastic moduli of the fluid-saturated composite rock are then obtained by applying one of the mixing laws commonly used to estimate the elastic properties of composite materials.

In the second scheme, the so-called Berryman-Milton scheme, the elastic moduli of the dry composite rock matrix are computed from the moduli of dry sand and clay matrices, using the same composite mixing law as used in the first scheme. The moduli of the saturated composite rock are then obtained using the equations of BROWN and KORRINGA (1975). These equations, together with the expressions for the coefficients derived by BERRYMAN and MILTON (1991), provide a rigorous extension of Gassmann equations to rocks with a heterogeneous solid matrix.

The moduli of the dry homogeneous sand and clay matrices are assumed to obey Krief's formula (KRIEF *et al.*, 1990) and as a mixing law we use the self-consistent coherent potential approximation proposed by BERRYMAN (1980a,b).

The compressional and shear velocities as functions of porosity and clay content for a given set of parameters depend on the distribution of total porosity between the sand and clay regions. If the distribution of porosity between sand and clay is relatively uniform, the predictions of the two schemes in the porosity range up to 0.3 are very similar. For higher porosities and medium-to-large clay content the elastic moduli predicted by the CG scheme are significantly higher than those predicted by the BM scheme.

This difference is explained by the fact that the BM model predicts the fully relaxed moduli, wherein the fluid can move freely between sand and clay regions. In contrast, the CG scheme predicts the no-flow or unrelaxed moduli. Our analysis reveals that due to the extremely low permeability of clays, at seismic and higher frequencies the fluid has no time to move between sand and clay regions and *vice versa*. Consequently the CG scheme is more appropriate for the frequencies used in geophysical exploration.

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