Wave propagation in double-porosity thermoelastic media

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ABSTRACT

The Lord-Shulman thermoporoelasticity theory couples the Biot and hyperbolic heat equations to describe wave propagation, modeling explicitly the effects of heat and fluid flows. We have extended the theory to the case of double porosity by taking into account the local heat flow (LHF) and local fluid flow (LFF) due to wave propagation. The plane-wave analysis finds the presence of the classical P and S waves and three slow P waves, namely, the slow (Biot) P_1 , the slow (Biot) P_2 , and a thermal slow P wave (or T wave). The frequency-dependent attenuation curves find that these slow waves manifest as Zener-like relaxation peaks, which are loosely related to the LFF, Biot, and LHF loss mechanisms. The viscosity and thermoelasticity properties can lead to the diffusive behavior of the three slow P modes. However, the S wave is considered to be independent of fluid and temperature influence. We examine the effect of thermophysical properties (e.g., thermal conductivity and relaxation time) on the wave velocity and attenuation of different modes. It is confirmed that the T wave is prone to be observed in media with high thermal conductivities and high homogeneity at high frequencies. Our double-porosity thermoelastic model reasonably explains laboratory measurements and well-log data from ultradeep fractured carbonates at high temperatures.

INTRODUCTION

The theory of thermoporoelasticity incorporates the equation of heat conduction into Biot's poroelasticity, coupling elastic deformations with temperature. The study of wave propagation in a thermoporoelastic medium has many applications in geophysics, such as geothermal exploration (e.g., Kana et al., 2015; Zhang et al., 2019), seismic attenuation (e.g., Armstrong, 1984; Pride et al., 2004), and seismicity (Rutqvist et al., 2013). In particular, the exploration and development of ultradeep hydrocarbon resources in Western China are usually associated with high-temperature environments (Fu, 2017). Wave propagation in such media involves the interaction among elastic deformation, heat flow, and fluid flow, and thus induces diffusive waves with frequency-dependent phase velocity and attenuation.

Classical thermoelasticity couples the heat flow and elastic deformation (Biot, 1956; Deresiewicz, 1957) but gives infinite slow P-wave (T-wave mode) velocities, especially at high frequencies (Savage, 1966; Achenbach, 1984). Lord and Shulman (1967) introduce thermal relaxation time to change the parabolic heat equation into hyperbolicity, which is consistent with the Maxwell-type mechanical model. Consequently, finite T-wave velocities are obtained. Therefore, the Lord-Shulman (LS) thermoelasticity has been extensively used for numerical simulations to investigate the effect of thermophysical properties on wave propagation in nonporous media (Carcione et al., 2018; Wei et al., 2020; Hou et al., 2021), predicting a classical P wave, a slow P diffusive wave (T mode), and an S wave. These two P-wave modes are similar to those of classical poroelasticity, with the difference that the thermal wave is related to heat (not fluid flow in poroelasticity). This latter mode is diffusive at low frequencies.

The LS thermoelasticity has been extended to the porous case, i.e., the so-called thermoporoelasticity, also predicting the classical Biot slow wave related to fluid flow (Noda, 1990; Sharma, 2008; Carcione et al., 2019; Baldonedo et al., 2020). The classical P-wave velocity is higher than the uncoupled case (isothermal case), but the S wave does not change with temperature (homogeneous media). With the Fourier pseudospectral method for numerical simulations, Carcione et al. (2019) show that the Boit mode to thermal mode conversion leads to mesoscopic energy attenuation. Wei et al. (2020) develop a frequency-domain Green's function as a displace-

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ment-temperature solution of thermoporoelasticity to investigate the effect of fluid viscosity and thermophysical properties.

Ba et al. (2011) combine the Biot theory of poroelasticity and Rayleigh's theory (BR) to develop a double-porosity theory. There are two types of pores that show an embedded relationship. The BR theory also can take into account the presence of two fluids (Ba et al., 2012). Zhang et al. (2021) apply the BR double-porosity theory to simulate wave propagation in infinituple-porosity media at multiple scales using the differential effective medium theory. In this work, we combine the LS and BR equations to obtain a thermoporoelasticity theory for double-porosity media. The proposed LS-BR theory predicts the fast P and S waves, as well as three slow modes: slow (Biot) P_1 , slow (Biot) P_2 , and thermal P (or T mode). The LS-BR theory can predict positive dispersions and quality factors, unlike the theories of Iesan and Quintanilla (2014) and Kumar et al. (2017).

The temperature dependence of dispersion and attenuation of elastic waves is essential to the study of the thermal effects in fluid-saturated rocks (Noda, 1990; Norris, 1994). Spencer (2013) analyzes this behavior in Ells River bitumen sands, showing that the P-wave moduli and attenuation vary by three orders of magnitude in frequency when increasing the temperature from 5°C to 48° C. Qi et al. (2021) show that different saturating fluids present different trends for the temperature- and frequency-dependent velocity and attenuation from 0°C to 50°C in carbonates. The proposed LS-BR model considers the thermal effects on the fluids and frame and their mutual interaction. We first investigate the effect of thermophysical properties (e.g., thermal conductivity and relaxation time) on the velocity and attenuation of various P modes. Then, we apply the LS-BR model to laboratory measurements and well-log data from ultradeep fractured carbonates at high temperatures.

EQUATIONS OF MOTION

Ba et al. (2011) propose a double-porosity medium for describing a material consisting of a solid skeleton and two fluid phases and explicitly determine the model parameters (six stiffness and five density coefficients) by using "Gedanken" experiments similar to the Johnson (1986). Based on the double-porosity model, we extend the thermoporoelasticity theory (single porosity) (Carcione et al., 2019) to include secondary pores and investigate how temperature affects attenuation.

Let us denote the displacement components of the frame by u_i and the displacement components of the double-porosity system (host medium and inclusions) by $U_i^{(m)}$ (m = 1, 2). The strain components of the frame and fluid can be defined, respectively, as

$$e_{ij} = \frac{1}{2} (\partial_j u_i + \partial_i u_j) \delta_{ij}, \quad \xi_{ij}^{(m)} = \frac{1}{2} (\partial_j U_i^{(m)} + \partial_i U_j^{(m)}) \delta_{ij},$$
(1)

where ∂_i is the partial derivative with respect to the spatial variable x_i and δ_{ij} is the Kronecker delta function. The dilatations are

$$e = e_{ii} = \partial_i u_i, \quad \xi_m = \xi_{ii}^{(m)} = \partial_i U_i^{(m)}, \tag{2}$$

where the Einstein summation of repeated indices is assumed.

Based on the heat transfer in the solid and fluid (Nield and Bejan, 2006), the thermal expansion coefficient of both phases can be introduced to include the inertial and the nonlinear advection terms in

the temperature equation (Carcione et al., 2019). The law of heat conduction for single-porosity media is

$$k\Delta T = \rho C_e(\dot{T} + \tau_0 \dot{T}) + T_0 \beta (\dot{e} + \tau_0 \ddot{e} + \dot{w}_{i,i} + \tau_0 \ddot{w}_{i,i}) + q,$$
(3)

where k is the coefficient of heat conduction (or thermal conductivity), Δ is the Laplacian operator, T is the increment of temperature above a reference absolute temperature T_0 for the state of zero stress and strain, C_e is the specific heat capacity, τ_0 is the relaxation time, a dot above a variable denotes time differentiation, w is the fluid filtration, q is a heat source, and

$$\beta = \beta_s + \left(1 - \frac{K_b}{K_s}\right)\beta_f,\tag{4}$$

where β_s and β_f are the coefficients of thermal stress (Carcione et al., 2019) for the solid and fluid, respectively, and other properties are given in Appendix A. It should be stressed that the relaxation time is crucial in the LS thermoelastic equations, but with few studies reported in the literature. It is the time taken for the acceleration of heat flow through the medium. Such a relaxation process prevents the instantaneous establishment of a temperature gradient in the medium. The existence of such a relaxation time has been attributed to the Maxwell-type mechanical model (Francis, 1972).

Extending equation 3 for heat conduction in double-porosity media, we have

$$k\Delta T = \rho C_e(\dot{T} + \tau_0 \ddot{T}) + T_0 \beta (\dot{e} + \tau_0 \ddot{e} + \sum_m (\dot{w}_{i,i}^{(m)} + \tau_0^{(m)} \ddot{w}_{i,i}^{(m)})) + q,$$
(5)

where the filtration of fluid *m* (Biot, 1962) is $w_i^{(m)} = \phi_m(U_i^{(m)} - u_i)$, ϕ_m denotes the porosities of the host medium (m = 1) and inclusions (m = 2), and $\tau_0^{(m)}$ is the thermal relaxation time of phase *m*. The composite density is $\rho = (1 - \phi)\rho_s + \rho_f \phi$ with ρ_s and ρ_f being the solid and fluid densities, respectively. In general, the pores within a representative volume are saturated by the same fluid, so we can assume that $\tau_0^{(1)} = \tau_0^{(2)} = \dots = \tau_0^{(m)} = \tau_0$. Thus, equation 5 reduces to

$$k\Delta T = \rho C_e(\dot{T} + \tau_0 \ddot{T}) + T_0 \beta (\dot{e} + \tau_0 \ddot{e} + \sum_m (\dot{w}_{i,i}^{(m)} + \tau_0 \ddot{w}_{i,i}^{(m)})) + q.$$
(6)

Because equation 6 involves only the volumetric strain and does not introduce shear strain, the S wave is not affected by temperature under the LS-BR theoretical framework. As shown by Ba et al. (2011), the constitutive relation of the double-porosity medium without local fluid flow (LFF) for the solid σ_{ij} and fluid $\tau^{(m)}$ is

$$\sigma_{ij} = (Ae + \sum_{m} Q_m \xi_m) \delta_{ij} + 2\mu e_{ij}, \quad \tau^{(m)} = Q_m e + R_m \xi_m,$$
(7)

where the stiffness coefficients (A, Q_1 , Q_2 , R_1 , and R_2) are given in Appendix A and μ is the shear modulus. If the thermal effects are

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considered, as in Carcione et al. (2019), we have the extended constitutive relations:

$$\sigma_{ij} = (Ae + \sum_{m} Q_m \xi_m - (\beta - \beta_f)T)\delta_{ij} + 2\mu e_{ij},$$

$$\tau^{(m)} = Q_m e + R_m \xi_m - \beta_f T.$$
 (8)

We use a scalar ς to describe the fluid variation during the LFF process,

$$\varsigma = \frac{1}{\phi_1} \left(1 - \frac{R_0^3}{R^3} \right),\tag{9}$$

where R_0 is the inclusion radius and R is the dynamic radius of the fluid sphere. Because of the conservation of fluid mass, we have (Ba et al., 2011)

$$\phi_1(\phi_2\varsigma) + \phi_2(-\phi_1\varsigma) = 0. \tag{10}$$

According to the kinetic energy function (Ba et al., 2011: equation 22), the kinetic energy can be expressed as

$$T' = \frac{1}{2}\rho_{00}\sum_{i}\dot{u}_{i}^{2} + \sum_{m}\rho_{0m}\sum_{i}\dot{u}_{i}\dot{U}_{i}^{(m)} + \frac{1}{2}\sum_{m}\rho_{mm}\sum_{i}(\dot{U}_{i}^{(m)})^{2} - \frac{1}{6}\frac{R_{0}^{2}\phi_{1}^{2}\phi_{2}\phi_{20}}{\phi_{10}}\rho_{f}\dot{\varsigma}^{2}, \quad (11)$$

where the density coefficients are

$$\rho_{00} = (1 - \phi)\rho_s - \rho_f \phi_1 (1 - \tau_1), \rho_{0m} = \rho_f \phi_m (1 - \tau_m),$$

$$\rho_{mm} = \rho_f \phi_m \tau_m, \rho = (1 - \phi)\rho_s + \rho_f \phi,$$
(12)

where the total porosity $\phi = \phi_1 + \phi_2$ and $\phi_m = v_m \phi_{m0}$, with v_m being the volume fraction of phase *m* and ϕ_{m0} being the porosity in each phase, and $\tau_m = 0.5((1/\phi_m) + 1)$ is the tortuosity.

Based on the frictional dissipation mechanism between the fluid and the frame, the dissipation function is similar to equations 30 and 33 of Ba et al. (2011):

$$D = -\frac{1}{2} \sum_{m} \phi_{m} \phi_{m0} \frac{\eta}{\chi_{m}} \dot{w}^{(m)} \cdot \dot{w}^{(m)} + \frac{1}{6} \left(\frac{\eta}{\chi_{1}}\right) R_{0}^{2} \phi_{1}^{2} \phi_{2} \phi_{20} \dot{\varsigma}^{2}.$$
(13)

Following Ba et al. (2011, 2012, 2013), we take partial derivatives of equations 11 and 13 with respect to the particle velocities of the solid and two fluid phases, and we introduce the constitutive relation (equation 8) into the momentum conservation (equation 11). According to the strain-displacement relations, the equation of motion is as follows:

$$\begin{split} &\mu u_{i,jj} + (A + \mu) u_{j,ij} + Q_1(U_{j,ij}^{(1)} + \phi_2 \varsigma) + Q_2(U_{j,ij}^{(2)} - \phi_1 \varsigma) + (\beta - \beta_f) T_{,i} \\ &= \rho_{00} \ddot{u}_i + \sum_m (\rho_{0m} \ddot{U}_i^{(m)} + b_m (\dot{u}_i - \dot{U}_i^{(m)})), \\ Q_1 u_{j,ij} + R_1(U_{j,ij}^{(1)} + \phi_2 \varsigma) + \frac{\phi_1}{\phi} \beta_f T_{,i} \\ &= \rho_{01} \ddot{u}_i + \rho_{11} \ddot{U}_i^{(1)} - b_1 (\dot{u}_i - \dot{U}_i^{(1)}), \\ Q_2 u_{j,ij} + R_2(U_{i,i}^{(2)} - \phi_1 \varsigma) + \frac{\phi_2}{\phi} \beta_f T_{,i} \\ &= \rho_{02} \ddot{u}_i + \rho_{22} \ddot{U}_i^{(2)} - b_2 (\dot{u}_i - \dot{U}_i^{(2)}), \\ kT_{i,i} = \rho C_e (\dot{T} + \tau_0 \ddot{T}) + T_0 \beta [(1 - \phi) (\dot{u}_{i,i} + \tau_0 \ddot{u}_{i,i}) \\ &+ \sum_m \phi_m (\dot{U}_{i,i}^{(m)} + \tau_0 \ddot{U}_{i,i}^{(m)})], \\ \phi_2(Q_1 u_{i,i} + R_1(U_{i,i}^{(1)} + \phi_2 \varsigma)) - \phi_1(Q_2 u_{i,i} + R_2(U_{i,i}^{(2)} - \phi_1 \varsigma)) \\ &= \frac{1}{3} R_0^2 \phi_1^2 \phi_2 \phi_{20} \left(-\frac{\rho_f}{\phi_{10}} \ddot{\varsigma} + \frac{\eta}{\chi_1} \dot{\varsigma} \right), \end{split}$$
(14)

where, following the Darcy law, η and χ_m represent fluid viscosity and skeleton permeability, respectively, and b_1 and b_2 are dissipation parameters given by

$$b_m = -\phi_m \phi_{m0} \frac{\eta}{\chi_m}.$$
 (15)

Equation 14 describes wave dispersion and attenuation in nonisothermal double-porosity media. It has 20 unknowns, i.e., five stiffnesses (A, Q_m , and R_m), five density coefficients (ρ_{00} , ρ_{0m} , and ρ_{mm}), two porosities ϕ_m , Darcy friction coefficients b_m , and thermal properties (k, τ_0 , T_0 , β , β_s , and β_f).

The proposed model is a straightforward extension of the doubleporosity model (Ba et al., 2011) and the thermoporoelasticity model (Carcione et al., 2019). If $\phi_2 = 0$, $\varsigma = 0$, and Q_2 , R_2 , ρ_{02} , ρ_{22} , and b_2 of the second type of pores are all zero, equation 14 reduces to the thermoporoelasticity equations (Carcione et al., 2019):

$$\mu u_{i,jj} + (A + \mu)u_{j,ij} + Q_1 U_{j,ij}^{(1)} + (\beta - \beta_f)T_{,i}$$

$$= \rho'_{00}\ddot{u}_i + \rho_{01}\ddot{U}_i^{(1)} + b'_1(\dot{u}_i - \dot{U}_i^{(1)}),$$

$$Q_1 u_{j,ij} + R_1 U_{j,ij}^{(1)} + \beta_f T_{,i} = \rho_{01}\ddot{u}_i + \rho_{11}\ddot{U}_i^{(1)} - b'_1(\dot{u}_i - \dot{U}_i^{(1)}),$$

$$kT_{i,i} = \rho C_e(\dot{T} + \tau_0 \dot{T}) + T_0\beta(\dot{u}_{i,i} + \tau_0 \ddot{u}_{i,i} + \dot{w}_{i,i}^{(1)} + \tau_0 \ddot{w}_{i,i}^{(1)}),$$

$$(16)$$

where $\rho'_{00} = \rho_{00} - \rho_{01}$ and $b'_1 = b_1/\phi_1$, although with a different notation (Carcione et al., 2019; Wei et al., 2020):

$$A = \lambda + M(\alpha^* - \phi_1)^2, \ Q_1 = \phi(\alpha^* - \phi_1)M, \ R_1 = \phi_1^2M, \ (17)$$

where these properties are given in Appendix A. If T = 0 and $\tau_0 = 0$, equation 14 reduces to the double-porosity equation of Ba et al. (2011):

$$\mu u_{i,jj} + (A + \mu)u_{j,ij} + Q_1(U_{j,ij}^{(1)} + \phi_2 \varsigma) + Q_2(U_{j,ij}^{(2)} - \phi_1 \varsigma)$$

$$= \rho_{00}\ddot{u}_i + \sum_m (\rho_{0m}\ddot{U}_i^{(m)} + b_m(\dot{u}_i - \dot{U}_i^{(m)})),$$

$$Q_1 u_{j,ij} + R_1(U_{j,ij}^{(1)} + \phi_2 \varsigma) = \rho_{01}\ddot{u}_i + \rho_{11}\ddot{U}_i^{(1)} - b_1(\dot{u}_i - \dot{U}_i^{(1)}),$$

$$Q_2 u_{j,ij} + R_2(U_{j,ij}^{(2)} - \phi_1 \varsigma) = \rho_{02}\ddot{u}_i + \rho_{22}\ddot{U}_i^{(2)} - b_2(\dot{u}_i - \dot{U}_i^{(2)}),$$

$$\phi_2(Q_1 u_{i,i} + R_1(U_{i,i}^{(1)} + \phi_2 \varsigma)) - \phi_1(Q_2 u_{i,i} + R_2(U_{i,i}^{(2)} - \phi_1 \varsigma))$$

$$= \frac{1}{3}R_0^2\phi_1^2\phi_2\phi_{20}\left(\frac{\rho_f}{\phi_{10}}\ddot{\varsigma} + \frac{\eta}{\chi_1}\dot{\varsigma}\right).$$

$$(18)$$

PLANE-WAVE ANALYSIS

We examine the characteristics of elastic waves in the doubleporosity thermoelastic medium by the following time-harmonic plane wave:

$$u_{i} = Bs_{i}e^{i\omega\left(t-\frac{l_{i}}{v_{c}}x_{j}\right)}, U_{i}^{(m)} = C_{m}d_{i}e^{i\omega\left(t-\frac{l_{i}}{v_{c}}x_{j}\right)},$$
$$T = De^{i\omega\left(t-\frac{l_{i}}{v_{c}}x_{j}\right)}, \varsigma = Ee^{i\omega\left(t-\frac{l_{i}}{v_{c}}x_{j}\right)},$$
(19)

`

where s_i and d_i are the polarizations of the solid and fluid particles, respectively; *B*, C_m , *D*, and *E* are the amplitudes; ω is the angular frequency; *t* is the time; v_c is the complex velocity; l_j and x_j denote the wave directions and the position component, respectively; and $i = \sqrt{-1}$.

Dispersion relations for frequency-dependent velocity and attenuation

Given equations 14 and 19, we have

$$\begin{split} & \left[\left(\frac{(b_1 + b_2)i}{\omega} - \rho_{00} \right) v_c^2 + \mu + (A + \mu) l_j l_i \right] Bs_j \\ & + \sum_m \left[\left(-\frac{b_m i}{\omega} - \rho_{0m} \right) v_c^2 + Q_m l_j l_i \right] C_m d_j \\ & - \left[\frac{(\beta - \beta_j)i}{\omega} v_c D + (Q_2 \phi_1 - Q_1 \phi_2) E \right] l_i = 0, \\ & \left[\left(-b_1 \frac{i}{\omega} - \rho_{01} \right) v_c^2 + Q_1 l_j l_i \right] Bs_j + \left[\left(b_1 \frac{i}{\omega} - \rho_{11} \right) v_c^2 + R_1 l_j l_i \right] C_1 d_j \\ & + \left(-\frac{\phi_1}{\phi} \beta_f \frac{i}{\omega} v_c D + R_1 \phi_2 E \right) l_i = 0, \\ & \left[\left(-b_2 \frac{i}{\omega} - \rho_{02} \right) v_c^2 + Q_2 l_j l_i \right] Bs_j + \left[\left(b_2 \frac{i}{\omega} - \rho_{22} \right) v_c^2 + R_2 l_j l_i \right] C_2 d_j \\ & - \left(\frac{\phi_2}{\phi} \beta_f \frac{i}{\omega} v_c D + R_2 \phi_1 E \right) l_i = 0, \\ & T_0 \beta(\tau_0 i \omega^3 + \omega^2) \left[(1 - \phi) Bs_i + \sum_m \phi_m C_m d_i \right] v_c l_i \\ & + \left[\rho C_e(i \omega + \tau_0 \omega^3) v_c + k \omega^2 \right] D = 0, \\ & (\phi_2 Q_1 - \phi_1 Q_2) B + R_1 \phi_2 C_1 - R_2 \phi_1 C_2 \end{split}$$

 $= \left[\frac{1}{3}R_0^2\phi_1^2\phi_2\phi_{20}\left(\frac{\rho_f}{\phi_{10}}i\omega - \frac{\eta}{\chi_1}\omega^2\right) - (\phi_2^2R_1 + \phi_1^2R_2)\right]ET_0.$ (20)

For S waves, the resulting dispersion relation is the case when $s_i l_i = d_i l_i = 0$, and

$$\mu \left(\frac{1}{v_c}\right)^2 = \rho_{00} - \frac{\mathbf{i}(b_1 + b_2)}{\omega} - \frac{\left(\rho_{01} + \frac{\mathbf{i}b_1}{\omega}\right)^2}{\rho_{11} - \frac{\mathbf{i}b_1}{\omega}} - \frac{\left(\rho_{02} + \frac{\mathbf{i}b_2}{\omega}\right)^2}{\rho_{22} - \frac{\mathbf{i}b_2}{\omega}}.$$
(21)

The S-wave complex velocity is not affected by the thermal properties.

For P waves, we have $s_i l_i = d_i l_i = 1$, and the longitudinal wave solutions can be expressed as a cubic equation in v_c^2 :

$$\begin{vmatrix} a_{11}v_c^2 + b_{11} & a_{12}v_c^2 + b_{12} & a_{13}v_c^2 + b_{13} & a_{14}v_c \\ a_{21}v_c^2 + b_{21} & a_{22}v_c^2 + b_{22} & a_{23}v_c^2 + b_{23} & a_{24}v_c \\ a_{31}v_c^2 + b_{31} & a_{32}v_c^2 + b_{32} & a_{33}v_c^2 + b_{33} & a_{34}v_c \\ a_{41}v_c & a_{42}v_c & a_{43}v_c & a_{44}v_c^2 + b_{44} \end{vmatrix} = 0,$$
(22a)

where

$$\begin{aligned} a_{11} &= (b_1 + b_2)\mathbf{i}/\omega - \rho_{00}, \ b_{11} &= [A + 2\mu + \mathbf{i}(Q_2\phi_1 - Q_1\phi_2)x_1]\omega^2, \\ a_{12} &= -b_1\mathbf{i}/\omega - \rho_{01}, \ b_{12} &= [Q_1 + \mathbf{i}(Q_2\phi_1 - Q_1\phi_2)x_2]\omega^2, \\ a_{13} &= -b_2\mathbf{i}/\omega - \rho_{02}, \ b_{13} &= [Q_2 + \mathbf{i}(Q_2\phi_1 - Q_1\phi_2)x_3]\omega^2, \\ a_{14} &= -(\beta - \beta_f)\mathbf{i}/\omega, a_{21} &= -b_1\mathbf{i}/\omega - \rho_{01}, \\ b_{21} &= [Q_1 - \mathbf{i}R_1\phi_2x_1]\omega^2, \\ a_{22} &= b_1\mathbf{i}/\omega - \rho_{11}, \ b_{22} &= [R_1 - \mathbf{i}R_1\phi_2x_2]\omega^2, a_{23} &= 0, \\ b_{23} &= -\mathbf{i}R_1\phi_2x_3\omega^2, a_{24} &= -\frac{\phi_1\beta_f\mathbf{i}}{\phi\omega}, \\ a_{31} &= -b_2\mathbf{i}/\omega - \rho_{02}, \ b_{31} &= [Q_2 + \mathbf{i}R_2\phi_1x_1]\omega^2, \\ a_{32} &= 0, \ b_{32} &= \mathbf{i}R_2\phi_1x_2\omega^2, a_{33} &= b_2\mathbf{i}/\omega - \rho_{22}, \\ b_{33} &= [R_2 + \mathbf{i}R_2\phi_1x_3]\omega^2, a_{34} &= -\frac{\phi_2\beta_f\mathbf{i}}{\phi\omega}, \\ a_{41} &= T_0\beta(1 - \phi)(\tau_0\mathbf{i}\omega^3 + \omega^2), \ a_{42} &= T_0\beta\phi_1(\tau_0\mathbf{i}\omega^3 + \omega^2), \\ a_{43} &= T_0\beta\phi_2(\tau_0\mathbf{i}\omega^3 + \omega^2), \ a_{44} &= \rho C_e(\mathbf{i}\omega - \tau_0\omega^2), \\ b_{44} &= k\omega^2, \end{aligned}$$

where

$$x_{1} = i(\phi_{2}Q_{1} - \phi_{1}Q_{2})/Z, x_{2} = i\phi_{2}R_{1}/Z, x_{3} = -i\phi_{1}R_{2}/Z,$$

$$Z = -\frac{1}{3}\omega\phi_{1}^{2}\phi_{2}\phi_{20}R_{0}^{2}(i\eta/\kappa_{1} + \omega\rho/\phi_{10}) - (\phi_{2}^{2}R_{1} + \phi_{1}^{2}R_{2}).$$
(22c)

Following Carcione (2014), we obtain the phase velocity, attenuation factor, and quality factor as follows:

$$v_p = \left[\operatorname{Re}\left(\frac{1}{v_c}\right) \right]^{-1}, \ \alpha = -\omega \operatorname{Im}\left(\frac{1}{v_c}\right), \ Q = \frac{\pi f}{\alpha v_p} = \frac{\operatorname{Re}(v_c)}{2\operatorname{Im}(v_c)},$$
(23)

where $f = \omega/(2\pi)$ is the frequency.

Examples

Examples for two different cases are calculated based on the material properties shown in Table 1. The equivalent skeleton moduli of two types of pores used for Figures 1 and 2 result from the same material properties of Ba et al. (2011) for weakly cemented sandstones. We follow Pride and Berryman (2003) to calculate the frame bulk modulus. We assign the volume ratios of two types of pores as largely different values (0.962 and 0.037, respectively) for such unconsolidated sandstones, with an attempt to neglect the interaction between two different inclusions. Figures 3-5 show two different sets of thermophysical properties (Carcione et al., 2019) to highlight the thermal effect on the elastic properties of viscoelastic porous media. Case 1 considers a typical water-saturated sandstone, whereas case 2 refers specifically to hypothetical material with high thermal conductivities and small relaxation time with an attempt to move the thermal attenuation peak into the frequency band of seismic data.

The propagation characteristics of elastic waves in porous media are usually affected by the coupling effect between temperature and deformation, such as tectonic seepage (Hamza and Chen, 2006) and gas mining (Wu, 2013), leading to a nonisothermal state generally. Therefore, we investigate the dispersion and attenuation properties with different ambient temperatures ($T_0 = 300^{\circ}$ K, 350° K, and 400° K). Figure 1 compares the results to those of Ba et al. (2011) double-porosity model (gray cross in Figure 1 by ignoring T_0 and T = 0). We see that the proposed LS-BR theory predicts three P waves and one T wave. The dispersion curve of the fast P wave has three inflection points, namely three attenuation peaks in Figure 1a and 1b. The strongest attenuation peak appears at low frequencies $(10^{1}-10^{2} \text{ Hz})$, corresponding to the mesoscopic energy loss caused by LFF. The higher frequencies $(10^4 - 10^5 \text{ Hz})$ show the effect of Biot's friction (Biot, 1962). The weakest relaxation peak at ultrasonic frequencies corresponds to the thermal attenuation. The P-wave velocity significantly increases by increasing ambient temperature (Figure 1a and 1c). The LFF attenuation peak and its characteristics are almost unaffected by the high ambient temperature (Figure 1b). However, the local heat flow (LHF) affects the Biotand the thermal-wave attenuation peaks, although the thermal-wave attenuation peak slightly increases. Considering the thermal effect, the Biot peak moves to low frequencies. The effect of temperature on the P2 wave is not significant because of the small number of cracks ($v_2 = 0.037$). Interestingly, the T wave shows decreasing phase velocity with increasing ambient temperature (Figure 2), i.e., an opposite trend compared with that of the P-wave mode.

The aforementioned two P-wave modes are similar to those of classical poroelasticity in that these modes propagate with slow velocities and are diffusive at low frequencies, but they become a high-speed weakly attenuated wave-like mode at high frequencies. From the point of view of physical nature, the slow P wave

Properties	Values for Figures 1 and 2	Values for Figures 3–5	
Grain bulk modulus K_s (GPa)	38	35	5
Shear modulus μ_s (GPa)	44	_	_
Density ρ_s (kg/m ³)	2650	2650	
Frame bulk modulus K_m (GPa)	_	1.7	
Shear modulus μ_m (GPa)	_	1.885	
Porosity of pore 1 ϕ_1	0.1	0.1	
Volume fraction of pore 1 v_1	0.963	0.0, 0.1, and 0.2	
Permeability of pore 1 χ_1 (D)	0.01	0.01	
Porosity of pore 1 ϕ_2	0.3	0.3	
Volume fraction of pore 2 v_2	0.037	1.0, 0.9, and 0.8	
Permeability of pore 2 χ_2 (D)	1	1	
Fluid density ρ_f (kg/m ³)	1040	1000	
Bulk modulus K_f (GPa)	2.5	2.4	
Viscosity η (Pa·s)	0.005	0.001	
Thermoelasticity coefficient β_f (kg/[m·s ² ·°K])	40,000	40,000	
Bulk specific heat capacity C_e (m ² /[s ² .°K])	0.38	0.38	
Thermoelasticity coefficient β (kg/[m·s ² ·°K])	120,000	120,000	
Absolute temperature T_0 (°K)	300	300	
		Case 1	Case 2
Thermal conductivity $k \text{ (m-kg/[s^3.°K])}$	1	10.5	4.5×10^{6}
Relaxation time τ_0 (s)	1.5×10^{-8}	1.5×10^{-8}	1.5×10^{-2}

Table	1.	Medium	properties.
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(Biot wave) is characteristic of global or macroscopic fluid flow driven by inertial forces in porous media (Müller et al., 2010; Deng and Morozov, 2019), whereas the thermal wave (T mode) is a global or macroscopic thermal diffusion propagation related to heat (not fluid flow).

Next, we follow Carcione et al. (2019) to consider two different cases to highlight the thermal effect on the elastic properties. Case 1 indicates a typical water-saturated sandstone, whereas case 2 refers specifically to a hypothetical material with high thermal conductivity and small relaxation time for moving the thermal attenuation peak into the frequency band of seismic data. In these models, the background is characterized by large pores (i.e., stiff pores) with the permeability $\chi_2 = 1$ D, whereas the embedded microcracks (i.e., compliant pores) have low porosities with the permeability $\chi_1 = 0.01$ D. The models reduce to a single-porosity thermoelastic system (Carcione et al., 2019) if the embedded cracks are removed. With all other parameters remaining the same, we consider different volume combinations of pores 1 and 2 to test the influence of volume fractions on the dispersion and attenuation of elastic waves.

Figures 3 and 4 show the frequency-dependent phase velocities and attenuation factors of fast P, slow P₂, and thermal P waves for the double-porosity (0.1 and 0.3) model with different volume fractions of pores 1 and 2, compared with the single-porosity (0.3) model (Carcione et al., 2019) for cases 1 and 2, respectively. Figure 5 compares the frequency-dependent phase velocities and attenuation factors of slow P₁ waves for the double-porosity (0.1 and 0.3) models of cases 1 and 2. The P-wave velocity gradually increases with decreasing porosities (Ba et al., 2012). When the



+++ Ba et al. (2011) — Regardless of the T_0 …… 300° K --- 350° K --- 400° K

Figure 1. (a, c, and e) Phase velocity and (b, d, and f) attenuation as a function of frequency for (a and b) fast P wave, (c and d) slow P wave (P_1) , and (e and f) slow P wave (P_2) , compared with Ba et al. (2011) double-porosity model. The properties are listed in Table 1.

rock contains only background pores ($v_1 = 0\%$ and $v_2 = 100\%$), the results predicted by equation 17 are the same as those of Carcione et al. (2019). In the case of double-porosity cases ($v_1 \neq 0\%$), the local flow losses appear, leading to two slow P waves (see Figure 3c–3f) and complex fast P anelasticity. For example, Figures 3a and 4a show two inflection points in a fast-P dispersion curve, which correspond to Biot and thermal attenuation peaks with their locations depending on the petrophysical and thermophysical properties of rocks.



Figure 2. (a) Phase velocity and (b) attenuation factor as a function of frequency for thermal wave. The properties are listed in Table 1.



Figure 3. Comparison of the (a, c, and e) frequency-dependent phase velocities and (b, d, and f) attenuation factors for (a and b) fast P, (c and d) slow P_2 , and (e and f) thermal P waves between the double-porosity (0.1 and 0.3) model, with different volume fractions of pores 1 and 2, and the single-porosity (0.3) model (Carcione et al., 2019) for case 1. The properties are given in Table 1.

Figure 3 shows that, for a given total porosity, the P-wave velocity does not decrease monotonically with increasing volume fractions in background pores. The fast-P velocity decreases initially followed by an increase with increasing crack contents, whereas the corresponding Biot attenuation peak increases gradually. Although the volume ratio of two types of pores affects the dispersion and attenuation of waves, it is very limited, especially for the two slow P and T waves (see Figures 3-5). Figure 4 shows the results for case 2 with similar trends to those in case 1, except that the T wave exhibits a negative velocity dispersion approximately 10⁴ Hz, but it does not mean that it violates the Kramers-Krönig relation. As discussed by Deng and Morozov (2018), there exist wave processes with negative phase velocity dispersion but satisfy the Kramer-Krönig relations with $Q^{-1} > 0$, for example, the surface waves in seismology (Aki and Richards, 2002). Frequency intervals of negative velocity dispersion also were observed by Winkler (1983) and theoretically predicted for cracked porous media by Jakobsen and Chapman (2009), and laboratory measurements on trabecular bone by Waters and Hoffmeister (2005).

Furthermore, we focus on the effects of thermal conductivity and relaxation time on wave properties. Thermal conductivity is one of the most important factors influencing wave propagation. Its values depend on the mineral composition of rocks and pore fillings. The relaxation time is related to the time of heat flow and determines the thermodynamic behavior of a substance in a nonstatic temperature field (Lotfy and Othman, 2011; Zhao and Lyu, 2015), especially in a high-temperature environment. The parameters used for the tests



Figure 4. Comparison of the (a, c, and e) frequency-dependent phase velocities and (b, d, and f) attenuation factors for (a and b) fast P, (c and d) slow P_2 , and (e and f) thermal P waves between the double-porosity (0.1 and 0.3) model, with different volume fractions of pores 1 and 2, and the single-porosity (0.3) model (Carcione et al., 2019) for case 2. The properties are given in Table 1.

are listed in Table 1, and the pore system consists of 10% pore 1 and 90% pore 2.

Figure 6 shows that, with increasing thermal conductivities, the fast P dispersion and the corresponding thermal attenuation increase from 10^{6} Hz, but almost remain unchanged below this frequency. In



Figure 5. Comparison of the (a and c) frequency-dependent phase velocities and (b and d) attenuation factors of slow P_1 waves for the double-porosity (0.1 and 0.3) model with different volume fractions of pores 1 and 2 for (a and b) case 1 and (c and d) case 2.



Figure 6. Comparison of the (a, c, e, and g) frequency-dependent phase velocities and (b, d, f, and h) attenuation factors of (a and b) fast P, (c and d) slow P_1 , (e and f) slow P_2 , and (g and h) thermal P waves for different thermal conductivities. The properties are listed in Table 1.



Figure 7. Comparison of the (a, c, e, and g) frequency-dependent phase velocities and (b, d, f, and h) attenuation factors of (a and b) fast P, (c and d) slow P_1 , (e and f) slow P_2 , and (g and h) thermal P waves for different relaxation times. The properties are listed in Table 1.

general, the thermal conductivity is positively correlated with the density of rocks (Gao et al., 2015). With increasing thermal conductivities, the phase velocity and attenuation peak of the fast P wave increase significantly. The slow P_1 and P_2 show a similar trend to the fast P wave. It is worth noting that, in Figure 6e and 6f, the diffusion frequency reduces and the corresponding attenuation peak moves to low frequencies with increasing thermal conductivities, possibly because of the decrease of viscosity.

Figure 7 shows the effect of relaxation time on individual wave modes, with similar characteristics as shown in Figure 6 associated with the effect of thermal conductivity. Classical thermoelasticity based on Fourier heat conduction is equivalent to the LS thermoelasticity with $\tau_0 = 0$, which indicates that a temperature perturbation at a given location can instantaneously be detected at some other location. This results in an infinite T-wave speed. Intuitively, $\tau_0 = 0$ is similar to a material with superconductivity. The relaxation time in a homogeneous medium can be defined as $k / C_e V^2$ (Rudgers, 1990) or $2\pi f_p$ (Wang and Santamarina, 2007), where V is the sound veloc-1 / ity and f_p is the peak frequency of T wave. We see that it is proportional to k, but inversely proportional to V^2 and f_p . Therefore, Figures 6 and 7 show an opposite trend of variations in dispersion and attenuation caused by thermal conductivity and relaxation time relative to V and f_{p} . With decreasing thermal conductivities, more relaxation time is needed to reach the steady-state, leading to a reduction of attenuation. Thus, the T wave is prone to be observed in the medium with high thermal conductivities at the high-frequency limit.

The presence of pore fluids significantly attenuates elastic waves in the frequency band of seismic data, especially contributing to the diffusive behavior of Biot waves, which is controlled by the viscosity of pore fluids. Figure 8 shows the frequency-dependent velocity and attenuation of these slow waves for three different fluids of hot water, ambient water, and oil, with their viscosities being 0.0002, 0.001, and 0.005 Pa·s, respectively. To highlight the influence of fluid viscosities on the local flow mechanism, we use the volume ratio of two types of

Figure 8. Comparison of the (a, c, e, and g) frequency-dependent phase velocities and (b, d, f, and h) attenuation factors of (a and b) fast P, (c and d) slow P₁, (e and f) slow P₂, and (g and h) thermal P waves for three pore fluids (hot water, ambient water, and oil) with different viscosities. The plot in the upper-right corner is for an enlarged view of the dashed box in (b). The properties are listed in Table 1.



pores: $v_1 = 0.963$ and $v_2 = 0.037$, as listed in Table 1 for Figures 1 and 2, but change the thermal conductivity to 5 m·kg/(s^{3.o}K).

From Figure 8b, we see that the fast-P attenuation due to the LFF presents three strong peaks which are concentrated around the band of 10¹-10³ and move toward low frequencies with increasing fluid viscosities. As shown in the upper-right corner of Figure 8 for an enlarged view, the second group of very weak attenuation peaks due to Biot dissipation mechanism occurs around the band of 10⁴-10⁶ Hz, only with the oil producing a peak-like attenuation. Because the ratio of fluid viscosity to permeability controls the behavior of Biot slow waves, the resulting attenuation peaks move toward high frequencies with increasing fluid viscosities. Meanwhile, we see that three different fluids have the same thermal peak induced by the LHF, implying that viscosity has no effect on the dispersion and attenuation of thermal waves that are controlled by thermophysical properties (τ_0 and k). It should be stressed that we use the same relaxation time for the porous medium saturated with different fluids, respectively. That is, the effect of viscosities on the equivalent relaxation time of the medium is assumed too small to be ignored.

As expected by the proposed double-pore thermoelasticity model, the S wave is independent of the LFF as well as the material's thermal properties. Figure 9 shows that the Biot loss is the only cause of S-wave attenuation. The influence of temperature on the S wave will be considered in future work.

APPLICATIONS TO EXPERIMENTAL AND LOGGING DATA

In this section, we apply the proposed doubleporosity thermoelastic model to laboratory measurements and well-log data. The rock samples and logging data are from ultradeep carbonate reservoirs in high-temperature environments. In these samples, the background material is characterized by large pores (i.e., stiff pores) with the volume fraction v_2 , whereas the embedded microcracks (i.e., compliant pores) have low porosities with the volume fraction v_1 . Figure 10 shows the workflow with two-step processes to create the temperature-dependent fluid model and the rock-skeleton model, respectively. The former involves three temperature-dependent fluid properties: $\rho_f(T)$, $\eta_f(T)$, and $V_f(T)$ (for details, see Appendix B). The latter contains two skeleton parameters: the microscopic pore structure $\phi_{20}(T)$ and the frame elastic moduli of the rock matrix. From these models, we can obtain the P-wave velocity in the temperature range of 0°C-300°C. We begin by estimating the pressure-dependent frame elastic moduli of the rock matrix. We calculate the temperature-dependent microcrack porosity ϕ_{20} . The fluid parameters are given by equations B-1 and B-2. In the next step, these parameters are incorporated into the five poroelasticity coefficients (Appendix A), five density coefficients (equation 12), and two dissipation coefficients (equation 15). Then, these coefficients and

thermophysical properties are substituted into the LS-BR model (equation 14) to obtain the temperature-dependent equations of motion. Finally, these equations are solved by plane-wave analysis to obtain the frequency-dependent velocity and attenuation of waterand oil-saturated rocks.

Experimental and logging data

As listed in Table 2 provided by Qi et al. (2021), the dolomite sample from Northwest China mainly consists of dolomites with a







Figure 10. Workflow for the proposed double-porosity thermoelastic model applied to experimental and logging data.

Table 2. Petrophysical properties of rock samples.

Sample	Dolomite	Carbonate
Porosity (%)	11.73	10.37
Permeability (mD)	0.138	1.430
Density (kg/m ³)	2510.00	2525.00
WS20 $V_{\rm P}$ (m/s)	5644.96	5754.17
WS140 $V_{\rm P}$ (m/s)	5489.68	5630.58
OS20 $V_{\rm P}$ (m/s)	5644.96	5726.02
OS140 V _P (m/s)	5454.76	5600.75

⁴WS20 $V_{\rm P}$ and WS140 $V_{\rm P}$: water-saturated P-wave velocities measured at 20°C and 140°C. ⁵OS20 $V_{\rm P}$ and OS140 $V_{\rm P}$: oil-saturated P-wave velocities measured at 20°C and 140°C. small amount of clay, whereas the carbonate sample from western Sichuan, China, is mainly composed of granular dolomites with dissolved pores. Both the samples are extracted from reservoirs at a depth of more than 4.5 km, with an in-situ temperature of approximately 140°C, and have similar mineral compositions, but they present very different petrophysical properties. The microstructures of the samples mainly include intergranular pores and microcracks (grain contacts). As indicated by Qi et al. (2021), ultrasonic measurements are performed with the central frequency of 1 MHz for all of the samples subject to a confining pressure of 80 MPa whereas the pore pressure is kept constant at 10 MPa. During the experiment, waveforms are first recorded at 20°C, and then the temperature is raised to 140°C and maintained for 30 min before the measurement at 140°C. The measured P- and S-wave velocities are listed in Table 2.

Logging data are from a well located in Tarim Basin, Northwest China, and correspond to Ordovician carbonate reservoirs (Qi, 2016) more than 7 km deep, with an in-situ temperature of approximately 160°C. The ultradeep reservoirs develop karst caves, pores, and microcracks caused by multistage strike-slip faulting and dissolution. As indicated by Wei et al. (2021), a carbonate sample extracted from the ultradeep reservoirs has 2.5% in porosity, 0.023 mD in permeability, and 2.66 g/cm³ in dry-rock density. There are no attenuation data and only P-wave impedance and $V_{\rm P}/V_{\rm S}$ ratio are used in this study.

Pressure- and temperature-dependent dry-rock elastic moduli

To interpret laboratory observations for the samples listed in Table 2, we follow Qi et al. (2021) to build up the pressure- and temperature-dependent empirical relations for the corresponding dryrock elastic moduli.

Measured by Qi et al. (2021) for the gas-saturated samples subject to a confining pressure of 80 MPa at ambient temperature, the dry-rock bulk and shear moduli are (71.24, 34.61) GPa and (77.34, 30.79) GPa for the dolomite and carbonate samples, respectively. In general, microcracks are assumed to be closed at high effective pres-



Figure 11. Temperature-dependent microcrack porosities of (a) water- and (b) oil-saturated samples subject to a confining pressure of 80 MPa.

sure (David and Zimmerman, 2012). Because the microcrack porosity is much smaller than the stiff background porosity, the total porosity at high pressure provides a good approximation to the stiff porosity (David and Zimmerman, 2012), and thus we can use the measured dry-rock elastic moduli at high effective pressure to estimate the frame moduli of host material with stiff pores.

As reported by Qi et al. (2021), microcracks can be affected by temperature variations, as shown in Figure 11 for the temperaturedependent porosities of water- and oil-saturated samples subject to a confining pressure of 80 MPa, whereas the pore pressure is kept constant at 10 MPa. Because there are only two measured values, we assume that the volume fraction of microcracks is linearly dependent on the temperature. We obtain the following temperaturedependence of microcrack porosity ϕ_{20} :

$$\phi_{20}^{\text{oil}} = 6.1594 \times 10^{-6}T + 0.003884,$$

$$\phi_{20}^{\text{water}} = 1.00242 \times 10^{-5}T + 0.00341,$$
 (24)

for the dolomite sample, and

$$\phi_{20}^{\text{oil}} = 7.60875 \times 10^{-6}T + 0.0033029,$$

$$\phi_{20}^{\text{water}} = 1.00241 \times 10^{-5}T + 0.0032029,$$
 (25)

for the carbonate sample.

We see that the temperature-dependent porosities of microcracks are different for different saturants, possibly because of the differential thermal properties of fluids. We use the Batzle-Wang (1992) empirical equation to obtain the fluid properties (see Appendix B).

Comparison with experimental and logging data

Based on the grain bulk and shear moduli (Qi et al., 2021) and the aforementioned dry-rock elastic moduli at high effective pressure, we can calculate the stiffness coefficients by Appendix A. Because both the samples listed in Table 2 mainly consist of dolomites, the thermophysical properties can be considered as $\beta = 150,000 \text{ kg/(m}\cdot\text{s}^{2}\cdot^{\circ}\text{K})$, $\beta_f = 50,000 \text{ kg/(m}\cdot\text{s}^{2}\cdot^{\circ}\text{K})$, $k = 3.62 \text{ m}\cdot\text{kg/(s}^{3}\cdot^{\circ}\text{K})$, and $C_e = 0.79 \text{ m}^2/(\text{s}^{2}\cdot^{\circ}\text{K})$. According to the petrophysical properties of the two samples, we consider $\chi_1 = 1.11 \times 10^{-3} \text{ mD}$, $\chi_2 = 1 \times 10^{-3} \text{ mD}$, and $R_0 = 2 \times 10^{-4} \text{ m}$ for both the samples. The aforementioned stiffness coefficients, thermophysical properties, and pore-structure parameters are substituted into the LS-BR model (equation 14) to obtain the temperature-dependent equations of motion. Finally, these equations are solved by plane-wave analyses to obtain the P-wave velocity and attenuation of water- and oil-saturated rocks.

Figure 12 shows the theoretical predictions under water- and oilsaturated conditions, respectively, which agree with experimental measurements. We see that the properties of pore fluids significantly affect the temperature dependence of P-wave velocities. It is worth mentioning that the P-wave velocity increases slightly with increasing temperature up to 50°C, but it decreases rapidly after 100°C. The initial phase with the P-wave velocity undergoing a gradual increase and then a rapid decrease seems very common for polymineralic rocks with differentiated thermophysical properties (Li et al., 2021). To distinguish between the primary and the secondary fluid properties, we change the fluid viscosity, fluid density, and microcrack porosity (i.e., fluid content), respectively, to see how individual properties affect the P-wave velocity. Figure 13 shows that increasing temperature reduces the viscosity and density, but it in-



Figure 12. Temperature-dependent P-wave velocities for (a) waterand oil-saturated dolomite and (b) carbonate samples.



Figure 13. Comparison of temperature-dependent P-wave velocities induced by the change of different fluid properties: (a) fluid viscosity, (b) fluid density, (c) and microcrack porosity.

creases the porosity. The microcrack closure seems to dominate the effects among these properties, implying that we could estimate the volume fraction of microcracks from observed data using the proposed model.

According to the properties at sonic frequencies for logging data, we follow Wei et al. (2021) to consider $\chi_1 = 1.11 \times 10^{-2}$ mD,



Figure 14. (a) Comparison of the predicted and logging P-wave velocities as a function of porosity for different temperatures, with (b) the resulting dissipation factors at sonic frequencies.

 $\chi_2 = 1 \times 10^{-2}$ mD, $\phi_{20} = 1\%$, and $R_0 = 2.5 \times 10^{-5}$ m. The petrophysical properties and elastic parameters (P-wave impedance and V_P/V_S ratio) are available in Wei et al. (2021), where variations in porosity demonstrate a good agreement with the distribution of P-wave impedances and V_P/V_S ratios. Figure 14a compares the predicted and logging P-wave velocities as a function of porosity for different temperatures, with the resulting dissipation factors for different temperatures displayed in Figure 14b as a function of porosity decreases with increasing porosity or temperature. Temperature gives a good constraint to the distribution of P-wave velocities.

CONCLUSION

We propose a double-porosity thermoelastic model to express the combined effects of local and global fluid flows and thermal effects on the wave velocity and attenuation. The isotropic medium consists of microcracks embedded in the porous background medium. Five waves propagate the classical P and S waves, the slow (Biot) P₁ wave, the slow (Biot) P2 wave, and a thermal P wave. The thermal wave of mode conversion causes additional energy dissipation, other than that due to fluid flow. These slow waves present the frequency-dependent attenuation that is related to the LFF, Biot, and LHF loss mechanisms, depending on the crack densities, fluid viscosities, and thermoelastic properties. Specifically, the viscosity/ permeability ratio determines the Biot slow wave (P1 and P2) behavior, whereas the thermal conductivity/specific heat ratio determines the thermal P-wave behavior. The proposed model allows for the analysis of P wave anelasticity in real rocks including the thermal effects with different frequencies at high temperature and pressure, as shown by comparing the theoretical results with laboratory and logging data.

The joint dispersion induced by the LFF and LHF occurs simultaneously in the nonisothermal thermoelastic media, but in different frequency ranges with different magnitudes, depending on the petrophysical and thermophysical properties of rocks. The LFF and LHF make the internal pressure and temperature of the system tend to steady-state, respectively, which needs a criterion related to the rock state to determine which is dominant. The analysis of this issue has not yet been reported in the literature and could be an extended research in the future. We study the propagation of waves in isotropic thermoelastic media by only considering one mechanism of compressional wave-heat interaction. Therefore, the S wave is independent of the temperature effects. Theoretically, temperature induces dilatations not shear, but grain-grain friction implies a change in temperature. This can be implemented by including shear strains into the heat equation, which is an extended research in the future. Carbonate samples are often anisotropic because of oriented fractures. The anisotropic double-porosity thermoelastic model could be established by incorporating the LS heat equation into an anisotropic double-porosity poroelastic model, which will be addressed in the near future.

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DATA AND MATERIALS AVAILABILITY

Data associated with this research are available and can be accessed via the following URL: Note: A digital object identifier (DOI) linking to the data in a general or discipline-specific data repository is strongly preferred.

APPENDIX A

STIFFNESS COEFFICIENTS OF THE BR THEORY

The stiffness coefficients of the BR theory are related to the physical properties of the grains and fluids, given by (Ba et al., 2011)

$$Q_{1} = \frac{\alpha^{*}(1 - \phi - K_{b}/K_{s})\phi_{1}K_{s}}{\alpha^{*}\left(1 - \phi - \frac{K_{b}}{K_{s}}\right)\phi_{1}K_{s} + \frac{K_{s}(\alpha^{*}\phi_{1} + \phi_{2})}{K_{f}}},$$

$$Q_{2} = \frac{\left(1 - \phi - \frac{K_{b}}{K_{s}}\right)\phi_{2}K_{s}}{1 - \phi - \frac{K_{b}}{K_{s}} + \frac{K_{s}(\alpha^{*}\phi_{1} + \phi_{2})}{K_{f}}},$$

$$R_{1} = \frac{(\alpha^{*}\phi_{1} + \phi_{2})\phi_{1}K_{s}}{\alpha^{*}\left(1 - \phi - \frac{K_{b}}{K_{s}}\right)\phi_{1}K_{s} + \frac{K_{s}(\alpha^{*}\phi_{1} + \phi_{2})}{K_{f}}},$$

$$R_{2} = \frac{(\alpha^{*}\phi_{1} + \phi_{2})\phi_{2}K_{s}}{1 - \phi - \frac{K_{b}}{K_{s}} + \frac{K_{s}(\alpha^{*}\phi_{1} + \phi_{2})}{K_{f}}},$$

$$A = (1 - \phi)K_{s} - \frac{2}{3}N - \frac{K_{s}}{K_{f}}(Q_{1} + Q_{2}),$$
(A-1)

where $\alpha^* = \xi_1/\xi_2$ and K_s and K_f are the grain and fluid bulk moduli, respectively. The dry-rock modulus K_b can be obtained from Qi et al. (2021). The parameter ϕ_m is the porosity of the phase *m*.

APPENDIX B

BATZLE-WANG EMPIRICAL EQUATIONS

The properties of pore fluids, including density, viscosity, and bulk modulus, are dependent on pressure and temperature. We use the Batzle and Wang (1992) empirical equations to obtain the properties at the temperature (T) and pore pressure (P_p).

The water density ρ_w is

$$\rho_{w} = \frac{(-80T - 3.3T^{2} + 0.00175T^{3} + 489P_{p} - 2TP_{p} + 0.016T^{2}P_{p} - 1.3 \times 10^{-5}T^{3}P_{p} - 0.333P_{p}^{2} - 0.002TP_{p}^{2})}{1 \times 10^{6}}.$$
(B-1a)

The sound speed of pure water V_w is

$$V_{w} = \sum_{i=0}^{4} \sum_{j=0}^{3} \varpi_{ij} T_{i} P_{p}^{j}, \qquad (B-1b)$$

where σ_{ij} are the constants given in Batzle and Wang (1992). The water viscosity η_w is

$$\begin{split} \eta_w &= 0.1 + 0.333S \\ &+ (1.65 + 91.9S^3) e^{-[0.2(S^{0.8} - 0.17)^2 + 0.45]T^{0.8}}, \quad \text{(B-1c)} \end{split}$$

where *S* is the weight fraction (ppm) of sodium chloride. The density of oil ρ_{oil} is

$$\rho_{\text{oil}} = \frac{\rho_0 + (0.00277P_p - 1.71 \times 10^{-7}P_p^3) \times (\rho_0 - 1.15)^2 + 3.49 \times 10^{-4}P_p}{0.972 + 3.81 \times 10^{-4} (T + 17.78)^{1.178}}$$
(B-2a)

where ρ_0 describes the reference density of petroleum, measured at the atmospheric pressure and 15.6°C.

The sound speed of oil V_{oil} is

$$V_{\text{oil}} = 2096 \left(\frac{\rho_0}{2.6 - \rho_0}\right)^{0.5} - 3.7T + 4.64P_p + 0.0115 [4.12(1.08\rho_0^{-1} - 1)^{0.5}]TP_p.$$
(B-2b)

Finally, the oil viscosity η_{oil} is

$$\begin{aligned} \eta_{\text{oil}} &= (10^{0.505 \times 10^{5.692 - 2.863/\rho_0 \times (17.8 + T)^{-1.163}} - 1) + 0.145P_p \\ &\times 10^{18.6 \times 0.11g\eta_0 + (lg\eta_0 + 2)^{-0.1} - 0.985}, \end{aligned} \tag{B-2c}$$

where η_0 is the viscosity measured at the atmospheric pressure.

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Biographies and photographs of the authors are not available.

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