On the group velocity of guided waves in drill strings

Flavio Poletto[a] and José M. Carcione
Istituto Nazionale di Oceanografia e di Geofisica Sperimentale (OGS), Borgo Grotta Gigante 42C, 34010 Sgonico, Trieste, Italy

(Received 5 September 2000; revised 8 January 2001; accepted 12 January 2001)

A key element to verify the drill-bit signal processing is the estimation of the time delay of the guided waves transmitted through the drill string from the bit to the surface. This delay can be measured from drill-bit vibrations or calculated using the mechanical properties of the drill string. The calculation of the complete solution and the exact dispersion relations requires expensive numerical modeling. In this work, a simple approach to compute the group velocity of extensional and torsional waves by averaging the drill-string properties in the low-frequency approximation is used. The result is a generalization of the group velocity for periodic systems in which stationary distribution of properties is assumed. Results obtained with real strings show that the group velocity in the bottom-hole assembly is greater than the velocity in a uniform rod and lower than the velocity in the drill pipes. © *2001 Acoustical Society of America.*
[DOI: 10.1121/1.1354196]

PACS numbers: 43.40.Cw, 43.40.At, 43.20.Mv [CBB]

I. INTRODUCTION

Drill-string guided waves contain information about the drilling conditions[1,2] and can be used to transmit data from the bit location to the surface. For instance, Rector and Marion[3] and Miranda *et al.*[4] investigated the potentials of these waves for while-drilling prediction ahead of the bit. The drill bit acts as a seismic source and, in addition, generates extensional and torsional vibrations in the drill string, whose wavelengths are much longer than the diameters of the drill string. These signals are used to correct the arrival time of the data acquired at the surface by a deployed seismic line. Because the correction is based on the group velocity of the guided waves, it is important to determine with accuracy this velocity, which depends on the drill-string acoustic properties and geometrical features.

Several authors investigated the behavior of guided waves in drill strings. Barnes and Kirkwood[5] analyzed the passband and stop band effects caused by the presence of tool joints in the drill pipes of an idealized (periodic) drill string. Drumheller[6,7] investigated the acoustic vibrations of the drill string for transmission of while-drilling information between the drill bit and the surface. Using the dispersion equation, Drumheller[6] calculated the group velocity of the extensional waves for different lengths and cross sections of the drill pipes and tool joints. He considered a periodic system. Using a full-wave modeling algorithm, Carcione and Poletto[8] solved the differential equations describing wave propagation through the drill string. They computed waveforms of the extensional, torsional, and flexural waves by modeling the geometrical features of the coupling joints, including piezoelectric sources and sensors.

In this letter, we propose a method to compute the group velocity of extensional and torsional waves in a drill string composed of several elements of arbitrary acoustic properties and geometrical characteristics. The method is based on averaging the forces, the mass, and the polar moment of inertia of the different components of the drill string. The condition of stationarity is required, that is, in a given length of drill string the proportion of each element (e.g., drill pipes, tool joints, etc.) remains constant. The resulting equations for the group velocity are not restricted to periodic systems.

II. ACOUSTIC PROPERTIES OF THE DRILL STRING

A drill string is composed of several sections and elements of different weights, diameters, and mechanical properties. It is divided into two main parts. The upper part, being the predominant one, is composed of several sections of drill pipes and coupling joints. The lower part is the bottom-hole assembly (BHA), which is heavier and of rather complex composition. The main parts of the BHA are the heavy-weight drill pipes and the drill collars. The drill string is normally made of steel with uniform elastic properties, but parts of the BHA are made of aluminum and hard rubber.

Each element of drill string is characterized by a density ρ, a length d, an internal radius r_i, an external radius r_e, a Young modulus E, and a shear modulus μ. Because at seismic frequencies the wavelengths of the guided waves are long compared to the drill-string lateral dimensions, the drill string can be approximated as an effective rod.

Let z be the axial coordinate of the drill string, u the relative axial displacement, and θ the relative angular displacement. The axial and angular strains are

$$\epsilon = \frac{\partial u}{\partial z} \quad \text{and} \quad s = \frac{\partial \theta}{\partial z}, \tag{1}$$

respectively. The strains (1) vary in the drill string according to the properties of the different elements. We assume the low-frequency approximation, which holds for wavelengths that are much longer than the length of the drill-string sections. At the low-frequency approximation, these sections can be seen as homogeneous rods, with constant diameter and elastic properties, where constant axial and torsional

[a]Electronic mail: fpoletto@ogs.trieste.it

stresses generate constant axial and angular strains, respectively.

If k is the wave number and ω is the angular frequency, the exact equations for the phase and group velocities are given by

$$v_p=\frac{\omega}{k} \quad \text{and} \quad v_g=\frac{\partial\omega}{\partial k}=\left(\frac{\partial k}{\partial\omega}\right)^{-1}, \tag{2}$$

where $\omega=\omega(k)$ [or $k=k(\omega)$] is the dispersion relation. Instead of using the exact dispersion relation, which would require a lengthy and unnecessary numerical calculation, we invoke the long-wavelength approximation and substitute the nonuniform drill string by an effective rod of similar average properties. Then, the group velocity at seismic frequencies can be easily calculated from these average properties.

We denote m_{ext} (in kg/m) and m_{tor} (in kg m) the mass per unit length and the polar moment of inertia per unit length, respectively. They are given by

$$m_{\text{ext}}=\rho A, \quad \text{and} \quad m_{\text{tor}}=\rho I, \tag{3}$$

where A is the cross section of the effective rod and I is its polar moment per unit mass and unit length, given by $I=(\pi/2)(r_e^4-r_i^4)$. Because in a uniform rod, the group velocities of extensional and torsional waves equal the respective phase velocities, that is, $v_{g(\text{ext})}=(E/\rho)^{1/2}$, and $v_{g(\text{tor})}\times(\mu/\rho)^{1/2}$,[9] the group velocities of the effective rod can be calculated as

$$V_{g(\text{ext})}=\sqrt{\frac{EA}{m_{\text{ext}}}}, \quad \text{and} \quad V_{g(\text{tor})}=\sqrt{\frac{\mu I}{m_{\text{tor}}}}, \tag{4}$$

where EA and μI are the average axial force per unit strain and the average torque per unit strain, respectively.

III. AVERAGE PROPERTIES

Let the drill string be composed of N elements, of axial lengths d_i and cross sections A_i, $i=1,...,N$. We calculate the average of a given property or field variable a by

$$\langle a\rangle=\left(\sum_{i=1}^{N} d_i\right)^{-1}\left[\sum_{i=1}^{N} d_i a_i\right]. \tag{5}$$

A. Average dynamical properties

The axial force (Newton's law) and the torque per unit length are,[9]

$$F_i=A_i\rho_i\frac{\partial^2 u_i}{\partial t^2}, \quad \text{and} \quad M_i=I_i\rho_i\frac{\partial^2\theta_i}{\partial t^2}, \quad i=1,...,N. \tag{6}$$

The average axial force and torque per unit lenght are then given by

$$F=\langle F_i\rangle, \quad \text{and} \quad M=\langle M_i\rangle, \tag{7}$$

respectively. We obtain the average masses $m_{\text{ext}}=A\rho$ and $m_{\text{tor}}=I\rho$ by assuming that the average axial force and torque are related to the average axial and angular displacements $\langle u_i\rangle$ and $\langle\theta_i\rangle$ by

$$F=A\rho\frac{\partial^2\langle u_i\rangle}{\partial t^2}, \quad \text{and} \quad M=I\rho\frac{\partial^2\langle\theta_i\rangle}{\partial t^2}. \tag{8}$$

Averaging Eq. (6) and using (7), (8), and $\partial^2\langle u_i\rangle/\partial t^2=\langle\partial^2 u_i/\partial t^2\rangle$ and $\partial^2\langle\theta_i\rangle/\partial t^2=\langle\partial^2\theta_i/\partial t^2\rangle$, we have

$$\left\langle A_i\rho_i\frac{\partial^2 u_i}{\partial t^2}\right\rangle=A\rho\left\langle\frac{\partial^2 u_i}{\partial t^2}\right\rangle,$$

and (9)

$$\left\langle I_i\rho_i\frac{\partial^2\theta_i}{\partial t^2}\right\rangle=I\rho\left\langle\frac{\partial^2\theta_i}{\partial t^2}\right\rangle.$$

Moreover, assuming statistical independence[10] between u_i and $A_i\rho_i$ and between θ_i and $I_i\rho_i$, we have

$$\left\langle A_i\rho_i\frac{\partial^2 u_i}{\partial t^2}\right\rangle=\langle A_i\rho_i\rangle\left\langle\frac{\partial^2 u_i}{\partial t^2}\right\rangle,$$

and (10)

$$\left\langle I_i\rho_i\frac{\partial^2\theta_i}{\partial t^2}\right\rangle=\langle I_i\rho_i\rangle\left\langle\frac{\partial^2\theta_i}{\partial t^2}\right\rangle$$

[for instance, since u_i, in the long-wavelength limit, varies linearly and very slowly from element to element, we may assume (omitting the time derivative) $\langle A_i\rho_i u_i\rangle=\langle A_i\rho_i\rangle\langle u_i\rangle$, even if $A_i\rho_i$ varies by large fractions from element to element].

Comparison of (9) and (10) yields

$$m_{\text{ext}}=A\rho=\langle A_i\rho_i\rangle, \quad \text{and} \quad m_{\text{tor}}=I\rho=\langle I_i\rho_i\rangle. \tag{11}$$

B. Average static properties

Let F be a constant axial force and M a constant torque acting on the drill string and let ϵ and s be the average axial and torsional strains, respectively. The strains are related to the axial force and torque by

$$\epsilon=\frac{F}{EA}, \quad \text{and} \quad s=\frac{M}{\mu I}, \tag{12}$$

where EA and μI are the averaged properties.

On the other hand, the average value of the axial and torsional strains are

$$\epsilon=\langle\epsilon_i\rangle, \quad \text{and} \quad s=\langle s_i\rangle, \tag{13}$$

respectively. Because

$$\epsilon_i=\frac{F}{E_iA_i}, \quad \text{and} \quad s_i=\frac{M}{\mu_iI_i}, \tag{14}$$

we obtain

$$\langle\epsilon_i\rangle=F\left\langle\frac{1}{E_iA_i}\right\rangle, \quad \text{and} \quad \langle s_i\rangle=M\left\langle\frac{1}{\mu_iI_i}\right\rangle. \tag{15}$$

Comparison of Eqs. (12) and (15) and use of (13) and (14) yields

$$EA=\left\langle\frac{1}{E_iA_i}\right\rangle^{-1}, \quad \text{and} \quad \mu I=\left\langle\frac{1}{\mu_iI_i}\right\rangle^{-1}, \tag{16}$$

respectively. Equations (16) are averages of the reciprocal of the acoustic properties. They are similar in form to the effective elasticity constant c_{33} of a thinly layered medium in the long-wavelength limit.[11]

TABLE I. Dimensions of the drill-collar elements.

System 1			System 2		
r_e (in.)	r_i (in.)	d (m)	r_e (in.)	r_i (in.)	d (m)
6.5	2.8125	0.47	8.0	2.8125	0.7
6.5	2.875	27.58	8.0	2.875	18.37
6.25	2.05	10.33	7.094	3.125	5.39
6.5	2.875	75.40	7.094	3.0	5.95
6.25	2.8125	1.89	8.0	2.875	55.44
6.5	2.875	18.65	8.0	2.75	2.0
6.75	2.8125	1.48	8.0	2.875	9.25
6.75	2.8125	1.48	8.0	2.8125	9.29
6.5	2.875	9.46	8.0	2.75	2.22
6.25	2.8125	0.66	9.25	2.875	0.3
8.5	0.1	0.25	9.625	2.0	9.64
			12.25	0.1	0.31

IV. GROUP VELOCITY

Substituting Eqs. (11) and (16) into (4) yields the group velocities for extensional and torsional waves in the long-wavelength (low-frequency) approximation

$$v_{g(\mathrm{ext})}=\left(\left\langle\frac{1}{E_iA_i}\right\rangle\langle A_i\rho_i\rangle\right)^{-1/2}=\left(\sum_{i=1}^{N} d_i\right)\left[\left(\sum_{i=1}^{N} d_i\,\rho_iA_i\right)\left(\sum_{i=1}^{N}\frac{d_i}{E_iA_i}\right)\right]^{-1/2}, \quad (17)$$

and

$$v_{g(\mathrm{tor})}=\left(\left\langle\frac{1}{\mu_iI_i}\right\rangle\langle I_i\rho_i\rangle\right)^{-1/2}=\left(\sum_{i=1}^{N} d_i\right)\left[\left(\sum_{i=1}^{N} d_i\,\rho_iI_i\right)\left(\sum_{i=1}^{N}\frac{d_i}{\mu_iI_i}\right)\right]^{-1/2}. \quad (18)$$

These equations are not restricted to periodic systems and provide a good approximation in the frequency range used for while-drilling investigations.

If the density and the Young modulus are constant, and the drill string is composed of a periodic system of pipes and coupling joints, Eq. (17) becomes the group velocity obtained by Drumheller.[7] Moreover, if the shear modulus is constant, Eq. (18) simplifies to

$$V_{g(\mathrm{tor})}=\sqrt{\frac{\mu}{\rho}}(d_1+d_2)\left[d_1^2+\left(\frac{I_1}{I_2}+\frac{I_2}{I_1}\right)d_1d_2+d_2^2\right]^{-1/2}, \quad (19)$$

where d_1 and d_2 and I_1 and I_2 are the lengths and polar moments of the pipes and coupling joints, respectively.

V. EXAMPLE

We consider wave propagation through the drill collars (part of the BHA) for two different cases. They are referred to as systems 1 and 2 in Table I, where d indicates the length of each element of the drill collar. In system 1, the radii are quite uniform, and in system 2 the radii show a larger variation. Each element has the same acoustic properties: $\rho=7840$ kg/m^3, $E=206$ GPa, and $\mu=78.5$ GPa. These values give rod velocities of 5126 and 3164.3 m/s for the extensional and torsional waves, respectively. The group velocities obtained from Eqs. (17) and (18) are 5123 and 3156.8 m/s and 5077 and 3044 m/s for the extensional and torsional waves corresponding to systems 1 and 2, respectively. As expected, due to uniform distribution of masses, the values for system 1 are close to those of the uniform rod.

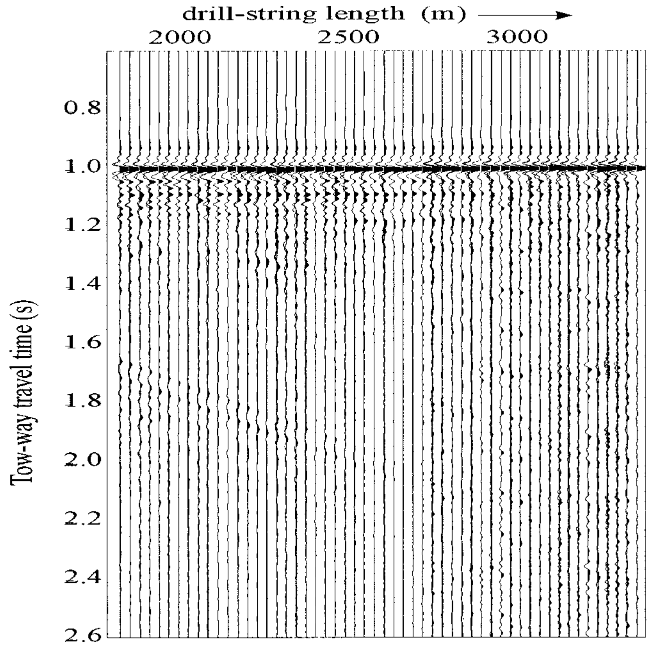

FIG. 1. Drill-bit axial pilot autocorrelation. Short- and long-period multiples have different group velocities in the BHA and in the drill pipes.

Let us consider now the drill pipe/coupling joint system above the BHA, with dimensions $r_e=5$ in., $r_i=4.275$ in., $d=9.2$ m (pipes), and $r_e=6.625$ in., $r_i=2.75$ in., $d=0.5$ m (tool joints). Assuming a periodic system, the extensional and torsional group velocities are 4727 and 2860 m/s, respectively. Note that the wave velocities in the drill collars for system 2 (with an uneven distribution of masses) are in between the velocities of a uniform rod and the velocities of the drill pipe/coupling joint system.

Figure 1 shows an example of real drill-bit signal autocorrelation corresponding to system 2 (see Table I). Each trace corresponds to approximately 30 min of listening time. These data are measured by an axial accelerometer on the top of the drill string, when drilling from 1900- to 3200-m depth. The direct arrival is aligned at 1 s. The dipping events are long period multiples of the drill string. Short-period reverberations are observable after the direct arrivals and after the long-period multiples. These events correspond to reverberations in the main parts of the BHA, i.e., drill collars (DC) and heavyweight pipes (HWDP). The analysis of the multiples in the DC gives a velocity of 5050 m/s, in good agreement with the calculated group velocity.

A similar analysis for the waves in the HWDP, assuming $E=206$ GPa, gives a velocity of 4740 m/s. Because of the different group velocities, the internal reverberations in HWDP and DC of equal length have different periods. The difference can be of 4 ms for a length of 150 m.

In summary, we obtained the group velocities for extensional and torsional waves in drill string at the long-wavelength (low-frequency) limit. For wavelengths longer than a characteristic length (for which the system is stationary), the drill string behaves like a uniform, or nearly uniform rod, whose mass is the average mass and whose elastic coefficients are algebraic combinations of the elastic coefficients of the single elements. The results are a generalization of well-known expressions for periodic systems, which are not expressed as averages, and therefore cannot suggest a generalization to nonperiodic media or media in which the acoustic and geometrical parameters and density can take more than two values. The results can be used to estimate the delays of low-frequency acoustic signals in the different drill-string sections.

ACKNOWLEDGMENTS

We thank ENI-Agip S.p.A. for funding the research and permission to publish the real data.

[1] J. D. MacPherson, J. S. Mason, and J. E. E. Kingman, ''Surface measurement and analysis of drill string vibrations while drilling,'' SPE/IADC 25777 (1993).

[2] M. Hutchinson, V. Dubinsky, and H. Henneuse, ''An MWD Downhole Assistant Driller,'' Society of Petroleum Engineers, Paper No. 30523 (1995).

[3] J. W. Rector III and B. P. Marion, ''The use of drill-bit energy as a downhole seismic source,'' Geophysics **56**, 628–634 (1991).

[4] F. Miranda, L. Aleotti, F. Abramo, F. Poletto, A. Craglietto, S. Persoglia, and F. Rocca, ''Impact of seismic while drilling technique on exploration wells,'' First Break **14**, 55–68 (1996).

[5] T. G. Barnes and B. R. Kirkwood, ''Passbands for acoustic transmission in an idealized drill string,'' J. Acoust. Soc. Am. **51**, 1606–1608 (1972).

[6] D. S. Drumheller, ''Acoustic properties of drill strings,'' J. Acoust. Soc. Am. **85**, 1048–1064 (1989).

[7] D. S. Drumheller and S. D. Knudsen, ''The propagation of sound waves in drill strings,'' J. Acoust. Soc. Am. **97**, 2116–2125 (1995).

[8] J. M. Carcione and F. Poletto, ''Simulation of stress waves in attenuating drill strings, including piezoelectric sources and sensors,'' J. Acoust. Soc. Am. **108**, 53–64 (2000).

[9] H. Kolsky, *Stress Waves in Solids* (Clarendon, Oxford, 1953).

[10] M. K. Ochi, *Applied Probability and Stochastic Processes* (Wiley, New York, 1990).

[11] J. M. Carcione, D. Kosloff, and A. Behle, ''Long wave anisotropy in stratified media: A numerical test,'' Geophysics **56**, 245–254 (1991).