ESTIMATING SEISMIC ATTENUATION (Q) IN THE PRESENCE OF RANDOM NOISE

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ABSTRACT

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The spectral-ratio and frequency-shift methods for estimating seismic attenuation (quality factor, Q) are reviewed and compared using seismic traces obtained from simulations in a 1D and 2D homogeneous constant-Q medium. In particular, we investigate the reliability of the two methods in estimating Q when random noise is added to the traces. The two methods are equivalent in the absence of noise. On the other hand, the spectral-ratio method performs better in the presence of noise. However, if the noise magnitude is evaluated from the frequency spectrum and compensated, the two methods are equivalent for low noise levels, while for very noisy data the frequency-shift method is more reliable. The comparison is performed for Q varying from 25 to 100 and noise-to-signal ratios from 0 to 100%.

KEY WORDS: quality factor, seismic modelling, spectral-ratio method, frequency-shift method.

INTRODUCTION

Seismic attenuation or quality factor (Q) has been recognised as a significant seismic indicator, which is not only useful for amplitude analysis and improving resolution, but also to obtain information on lithology, saturation, permeability and pore pressure (Best et al., 1994; Dasgupta and Clark, 1998; Carcione et al., 2003, Helle et al., 2003, Carcione and Picotti, 2006). Hence,

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estimation of seismic attenuation is as important as the estimation of interval velocities (Rossi et al., 2005), and provides additional information for rock characterisation and reservoir studies. On the other hand, high frequency losses due to absorption reduce the seismic bandwidth, and consequently the resolution of the seismic images. In this case, attenuation is regarded as a disturbance which must be eliminated by inverse Q-filters (Wang, 2003). So, while in one case we need to determine attenuation, in the other case we have to compensate for it. In both cases, the main problem is to obtain reliable Q estimations. Efforts to estimate Q often result in undesirable consistent errors for many reasons (Haase and Stewart, 2004). The main difficulty is the fact that, due to the low frequency content of seismic waves, the attenuation effects are usually small, and generally they can be measured with accuracy only for long distances. Tonn (1991) investigated several methods for the computation of attenuation, and showed that generally no single method is superior to the other: depending on the specific situation, some methods are more suitable than others. However, many of the uncertainties in Q estimation are often caused by the use of simplified attenuation models. All methods based on amplitude decay, for example, suffer when unity transmission coefficients are assumed, because amplitudes may be affected by many different factors, as the reflection/transmission phenomena. In this case, effective quality factors are estimated, which are different to intrinsic Q. On the contrary, the methods based on the evaluation of the frequency-centroid shift are not affected by reflection/transmission losses, but as we show in this work, they are more sensitive to the presence of random noise. In this paper, we used two analytical computational procedures to model the seismic response of a constant-Q medium: the first, based on the Green's function, is used for 1D simulations (Kjartansson, 1979) while the second, based on fractional derivatives, is used for 2D simulations (Carcione et al., 2002). Plane P waves are modelled in an homogeneous, isotropic and anelastic medium, under the assumption of a frequency-independent Q. Then, we revisit the spectral-ratio and frequency-shift techniques for the intrinsic Q-factor estimation and apply them to the synthetic seismic traces, including different levels of random noise. Finally, we compare the two methods and show a simple technique to compensate for the random noise.

BASIC THEORY

Phase and group velocities in a constant-Q model

Intrinsic attenuation is the process by which wave energy is transformed to heat, resulting in a reduction of the bandwidth of the signal. To model this phenomena, we need to understand how the velocity of the seismic signal depends on Q. Generally, the complex bulk modulus is a function of frequency and it is given by (Carcione et al., 2002)

$$M(\omega) = M_0(i\omega/\omega_0)^{2\gamma} , \tag{1}$$

where M_0 is the relaxed modulus, ω_0 is a reference angular frequency and γ is a dimensionless parameter. The concept of complex velocity is very useful to define phase and group velocities. The complex velocity is given by

$$v = \sqrt{(M/\rho)} , \tag{2}$$

where ρ is the density. Then, the phase velocity (Carcione, 2001) is

$$c = [\mathrm{Re}(1/v)]^{-1} = c_0|\omega/\omega_0|^\gamma , \tag{3}$$

where

$$c_0 = \sqrt{(M_0/\rho)}[\cos(\pi\gamma/2)]^{-1} \tag{4}$$

is the phase velocity at $\omega = \omega_0$. The parameter γ can be expressed in terms of Q by the following equation

$$\gamma = (1/\pi)\tan^{-1}(1/Q) . \tag{5}$$

It follows that Q is independent of frequency (Q is constant), and we see that $Q > 0$ is equivalent to $0 < \gamma \leq ½$. The complex wavenumber is defined by

$$k = \omega/v = \kappa - i\alpha , \tag{6}$$

where κ is the real wavenumber and α is the attenuation factor (Carcione et al., 2002):

$$\alpha = -\omega \mathrm{Im}(1/v) = \tan(\pi\gamma/2)\mathrm{sgn}(\omega)\omega/c . \tag{7}$$

The relation between the attenuation factor and the intrinsic quality factor Q can be expressed as (Carcione, 2001, p. 139)

$$\alpha = (\omega/c)[\sqrt{(Q^2 + 1)} - Q] \approx (\pi/cQ)f = \xi f , \tag{8}$$

where $f = \omega/2\pi$ is the frequency, ξ is the attenuation coefficient and the second relation in the right-hand side is valid for large values of Q (low-loss solid). Since there is physical evidence that attenuation in rocks is almost linear with frequency (Bland, 1960; Futterman, 1962; Kjartansson, 1979), constant-Q models provide a good approximation.

On the other hand, the group velocity is

$$v_g = [\mathrm{Re}(dk/d\omega)]^{-1} = [\mathrm{Re}(a/v)]^{-1} , \tag{9}$$

where k is the complex wavenumber and the complex coefficient a is given by (Ben-Menahem, 1981; Carcione, 2001)

$$a = 1 - (\omega/v)(dv/d\omega) = 1 - \gamma \ . \quad (10)$$

Hence, for a lossless ($Q = \infty$) and isotropic medium, we have from (5) that $\gamma = 0$ and all the velocities coincide. In that case, the velocity of the energy of the pulse is identical to the group velocity (Felsen and Marcuvitz, 1973; Mainardi, 1987). Carcione (1994) shows that the concept of seismic group velocity as the velocity of the energy is lost at high attenuation. Therefore, generally, for an absorbing and isotropic medium phase and energy velocity coincide and differ from the group velocity.

1D and 2D modelling

We consider a viscoacoustic isotropic absorbing medium and a Ricker-type wavelet source whose time-history is

$$S(t) = \exp[-\Delta\omega^2(t - t_0)^2/4]\cos[\omega_0(t - t_0)] \ , \quad (11)$$

and whose frequency spectrum is

$$S(\omega) = (\sqrt{\pi}/\Delta\omega)\Big\{\exp[-\{(\omega + \omega_0)/\Delta\omega\}^2] + \exp[-\{(\omega - \omega_0)/\Delta\omega\}^2]\Big\}\exp(i\omega t_0) \ , \quad (12)$$

where t_0 is a delay time, ω_0 is the central angular frequency and $\omega_{max} = 2\Delta\omega$ is the width of the pulse ($\Delta\omega = \omega_0$ in this work). The power spectral density $\Phi(\omega) = S(\omega)S^*(\omega)$ describes how the energy of the signal is distributed with frequency and it is related to the total energy in the time domain by Parseval's theorem, which states that the area under the spectral density curve is equal to the area under the square of the magnitude of the signal. This gives the following expression for the root mean square (RMS) amplitude of the source signal:

$$S_{RMS} = \sqrt{\{(1/T_{max}) \int_0^{T_{max}} S^2(t)dt\}} = \sqrt{\{(1/2\pi T_{max}) \int_0^{\omega_{max}} \Phi(\omega)d\omega\}} \ , \quad (13)$$

where T_{max} is the pulse length.

The frequency-domain response of the 1D medium is given by (e.g., Eckart, 1948):

$$U(x,\omega) = F(\omega)G(x,\omega) = F(\omega)\exp(-ikx) \ , \quad (14)$$

where $G(x,\omega)$ is the Green's function (impulse response) of the medium and x is the travel distance. The inverse Fourier transform of $U(x,\omega)$ gives the response in the time domain.

Let us now consider the propagation of the pulse in a 2D medium. The 2D wave equation can be written in the form

$$\partial^\beta u/\partial t^\beta = b[(\partial^2 u/\partial x^2) + (\partial^2 u/\partial z^2)] \quad , \tag{15}$$

were $u(x,z,t)$ is a field variable (Carcione, 2001). Considering a plane wave

$$u(x,z,t) = \exp[i(\omega t - k_x x - k_z z)] \quad , \tag{16}$$

we obtain the dispersion relation

$$(i\omega)^\beta + bk^2 = 0 \quad , \tag{17}$$

where (k_x,k_z) is the complex wavevector and $k = \sqrt{(k_x^2 + k_z^2)}$ is the complex wavenumber. Eq. (17) is the Fourier transform of eq. (15), and allows the calculation of the phase velocity corresponding to each Fourier component. The properties of the Fourier transform when it acts on fractional derivatives are well established, and a rigorous treatment is available in the literature (e.g., Dattoli et al., 1998). Comparison of eqs. (17), (6) and (1) gives

$$\beta = 2 - 2\gamma \quad , \quad b = (M_0/\rho)\omega_0^{-2\gamma} \quad . \tag{18}$$

Equation (15) with the parameters b and β defined by the relations (18) is the wave equation corresponding to Kjartansson's stress-strain relation (Kjartansson, 1979). In order to obtain realistic values of the quality factor, corresponding to wave propagation in rocks, $\gamma \ll 1$ and the time derivative in eq. (15) has a fractional order. We solved this equation numerically, using a finite-difference algorithm based on the Grünwald-Letnikov and central-difference approximations (Carcione et al., 2002), which are extensions of the standard finite-difference operators for derivatives of integer order (Letnikov, 1868; Gorenflo, 1997). Unlike the standard operator of differentiation, the fractional operator increases in length as time increases, since it must keep the memory effects. However, after a given time period the operator can be truncated (short memory principle).

Q-estimation techniques

We use two approaches to estimate attenuation: the classical spectral-ratio method and the frequency-shift method. The spectral-ratio approach (e.g., Dasgupta and Clark, 1998) uses the property that for frequency-independent

Q in the bandwidth of interest, a seismic wavelet will have its spectral amplitude S(f) modified to R(f) after travelling along a ray path from the source to the receiver. It is based on the equation (Quan and Harris, 1997)

$$\int_{ray} \xi dl = (1/f)\ln[GS(f)/R(f)] \ , \tag{19}$$

where ξ is the attenuation coefficient defined in (8) and G lumps many complicated processes together (such as scattering, geometrical spreading, source and receiver coupling, radiation patterns and reflection/transmission effects), and it is difficult to determine. However, if we consider a wave travelling along a distance x in an homogeneous medium, the amplitudes are only affected by the geometrical spreading and eq. (19) can be written as follows (Carcione et al., 2003):

$$\ln[S(f)/R(f)] = \xi xf + \ln(G) \ . \tag{20}$$

Hence, plotting the logarithm of the spectral ratio as a function of the frequency should yield a linear trend whose slope, p, is a function of Q. Then:

$$Q = \pi x/pc \ . \tag{21}$$

A major strength of this approach is that any frequency-independent scaling factor, as the geometrical spreading, falls into the intercept term of the linear regression and does not affect the Q estimation.

The frequency-shift approach is based on the fact that, as the wavelet propagates within the medium, the high frequency part of the spectrum decreases faster than its low frequency part. As a result, the centroid of the signal spectrum is downshift from f_S to a lower frequency f_R after the propagation from the source to the receiver. Under the assumption of a constant-Q model, this downshift $\Delta f = f_S - f_R$ is proportional to a linear integral of the attenuation along the ray path (Quan and Harris, 1997)

$$\Delta f = \sigma_S^2 \int_{ray} \xi dl \ , \tag{22}$$

where σ_S is the variance of the source. Hence, if the wave propagates a distance x in an homogeneous medium , the quality factor is given by:

$$Q = \pi x \sigma_S^2 / c\Delta f \ . \tag{23}$$

Since amplitudes are easily affected by many factors, if the signal bandwidth is broad enough and the attenuation is high enough to cause noticeable losses of high frequencies during the propagation, the frequency-shift method appears to be more reliable than the spectral-ratio method. Moreover, the integral eq. (22) is analogous to the equation that relates the seismic velocities to the measured traveltimes. Hence, it can be used for tomographic inversion and easily implemented in algorithms based on the ray-tracing method (Rossi et al., 2005).

EXAMPLES

1D simulations

In this first example we consider an homogeneous 1D medium, and a Ricker-type wavelet source whose frequency spectrum is described by eq. (12), with a reference frequency $f_0 = \omega_0/(2\pi) = 150$ Hz and a cut-off frequency $f_{max} = 2f_0 = 300$ Hz. The source is located at the origin $x = 0$ and the receiver is located at 250 m from the source. The phase velocity corresponding to the dominant frequency is $c_0 = 2000$ m/s, and we test four values of Q: 25, 50, 75 and 100. Fig. 1 shows the time propagation (a) and frequency spectra (b) corresponding to the case Q = 50. The error (or deviation) in the evaluation of Q is given as the percentage difference between the calculated quality factors, by using the eqs. (21) and (23), and the true quality factors. In all the cases, if there is no noise, the deviations obtained by using the frequency-shift method is less than 0.3%. Using the spectral ratio method the deviations are quite higher, but the two methods become equivalent if we remove the frequency components outside the interval $[\min(f_S - \sigma_S, f_R - \sigma_R), \max(f_S + \sigma_S, f_R + \sigma_R)]$. Then, we add to the seismic traces different noise levels, calculated as a percentage (noise-to-signal ratio NSR) of the RMS amplitude S_{RMS} of the source function. In our example, using eq. (13), $S_{RMS} = 0.333$ and the amplitude variance of the added random noise is defined as $\sigma_N = (S_{RMS} \cdot NSR)/100$. Fig. 2 shows the time propagation (a) and frequency spectra (b) corresponding to Q = 50 and NSR = 50%. We computed the quality factor Q by adding different percentages of random noise to the seismic traces: NSR = 25%, 50%, 75% and 100%. Since this is a stochastic experiment, for each value of NSR we iterated the Q evaluation procedure for 500 times. The deviation corresponding to each case is the percentage difference between the true Q value and the average calculated Q. Fig. 3 shows the deviations corresponding to Q = 50 (a) and Q = 100 (b). We notice that, when NSR > 25%, the spectral-ratio method is definitely better than the frequency-shift method. This is because we ignore the presence of random noise in the computation. However, for each method, when NSR > 50%, the errors are unacceptable high. It is possible to reduce considerably the errors by evaluating the average amplitude of the random noise A_N from the amplitude spectra. We computed A_N in a frequency range from f_{max} to $2f_{max}$, and redefined the centroid frequency f_S and variance σ_S of the source signal S(f) as follows:

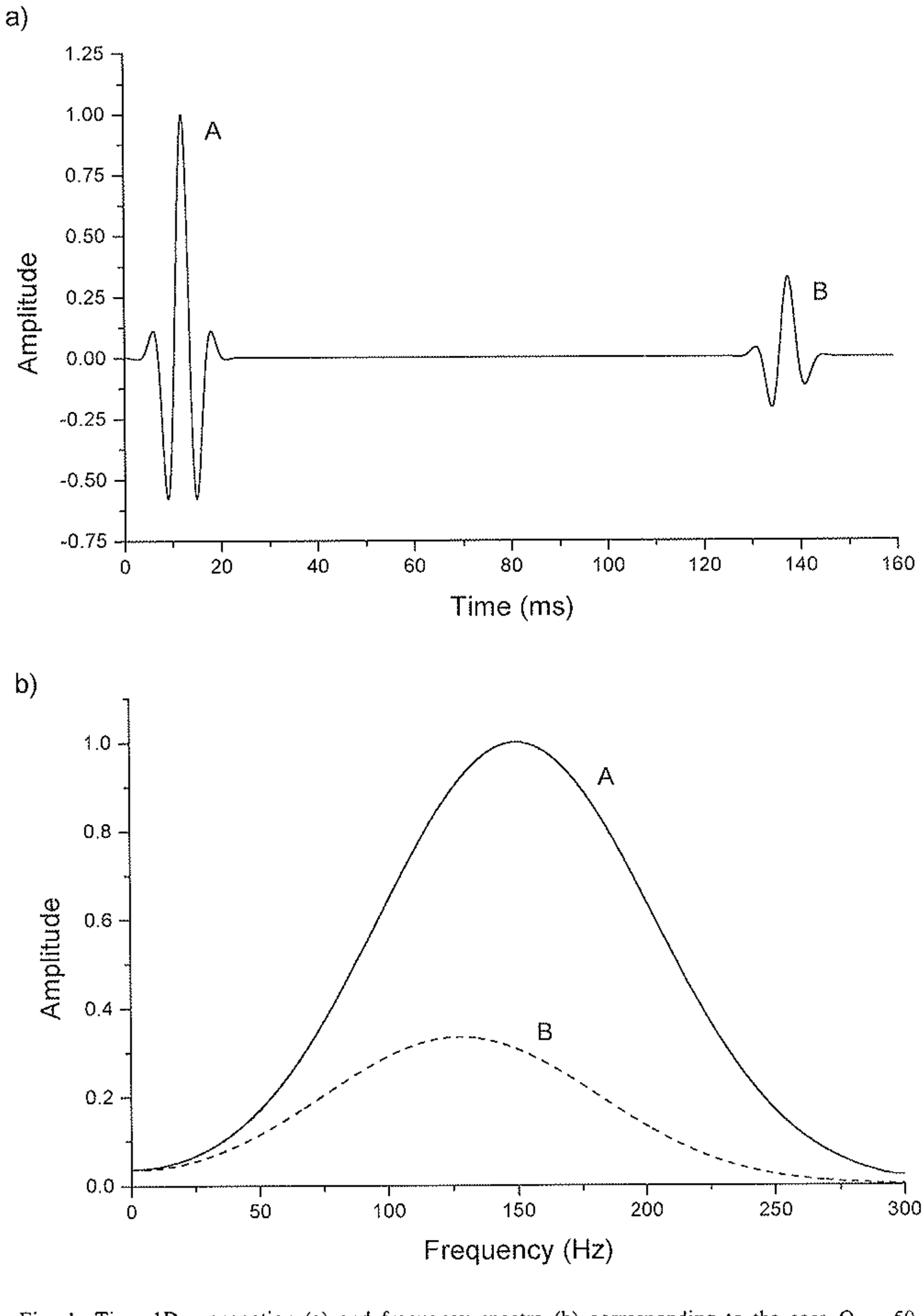

Fig. 1. Time 1D propagation (a) and frequency spectra (b) corresponding to the case Q = 50 without random noise, where A indicates the source and B the output signal. The reference frequency is $f_0 = \omega_0/(2\pi) = 150$ Hz, the offset is x = 250 m and the phase velocity corresponding to the dominant frequency is $c_0 = 2000$ m/s.

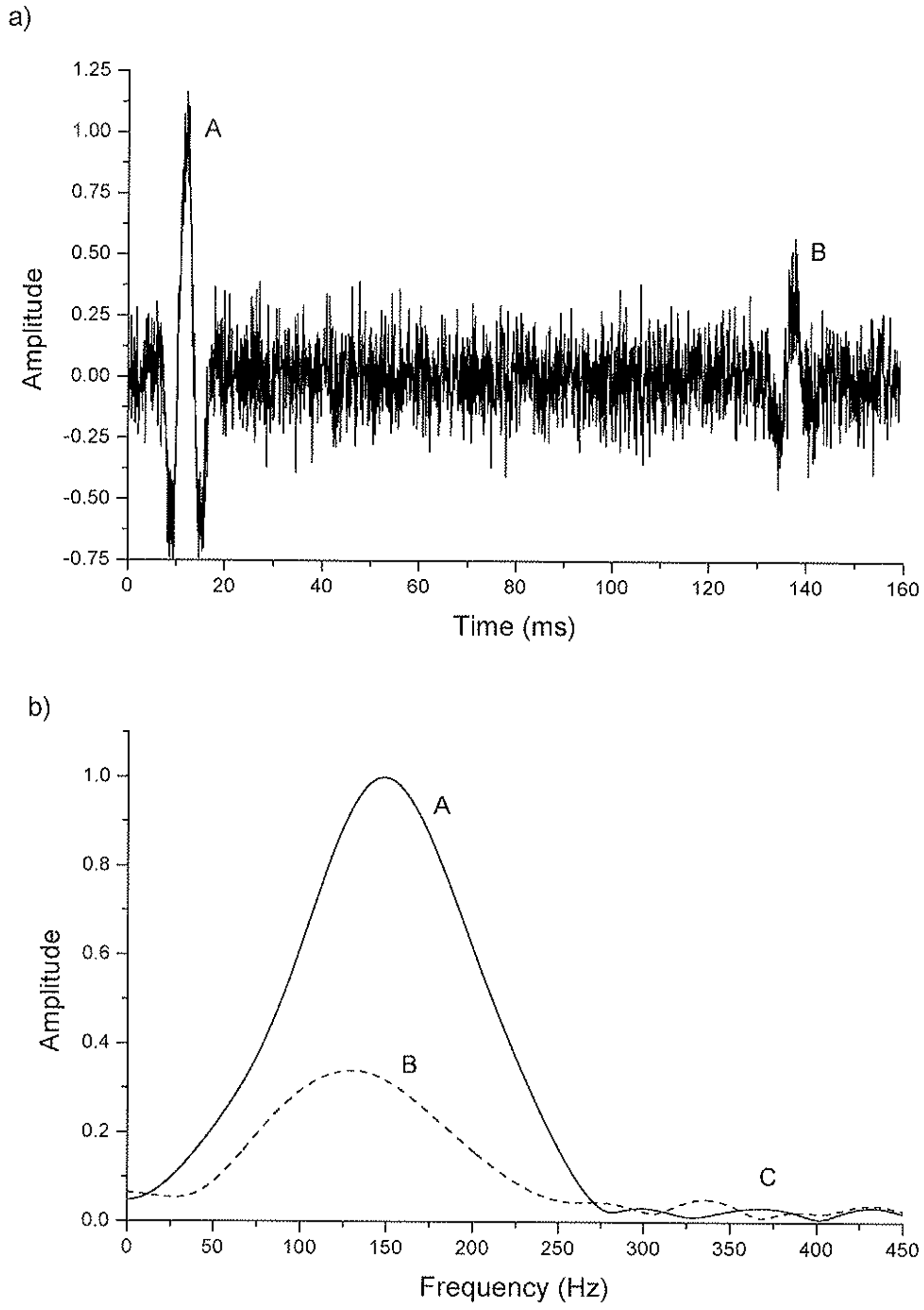

Fig. 2. Time 1D propagation (a) and frequency spectra (b) corresponding to the case Q = 50 and NSR = 50%, where A indicates the source and B the output signal. The ripples C outside the signal frequency band are due to the presence of random noise. The reference frequency is $f_0 = \omega_0/(2\pi)$ = 150 Hz, the offset is x = 250 m and the phase velocity corresponding to the dominant frequency is c_0 = 2000 m/s.

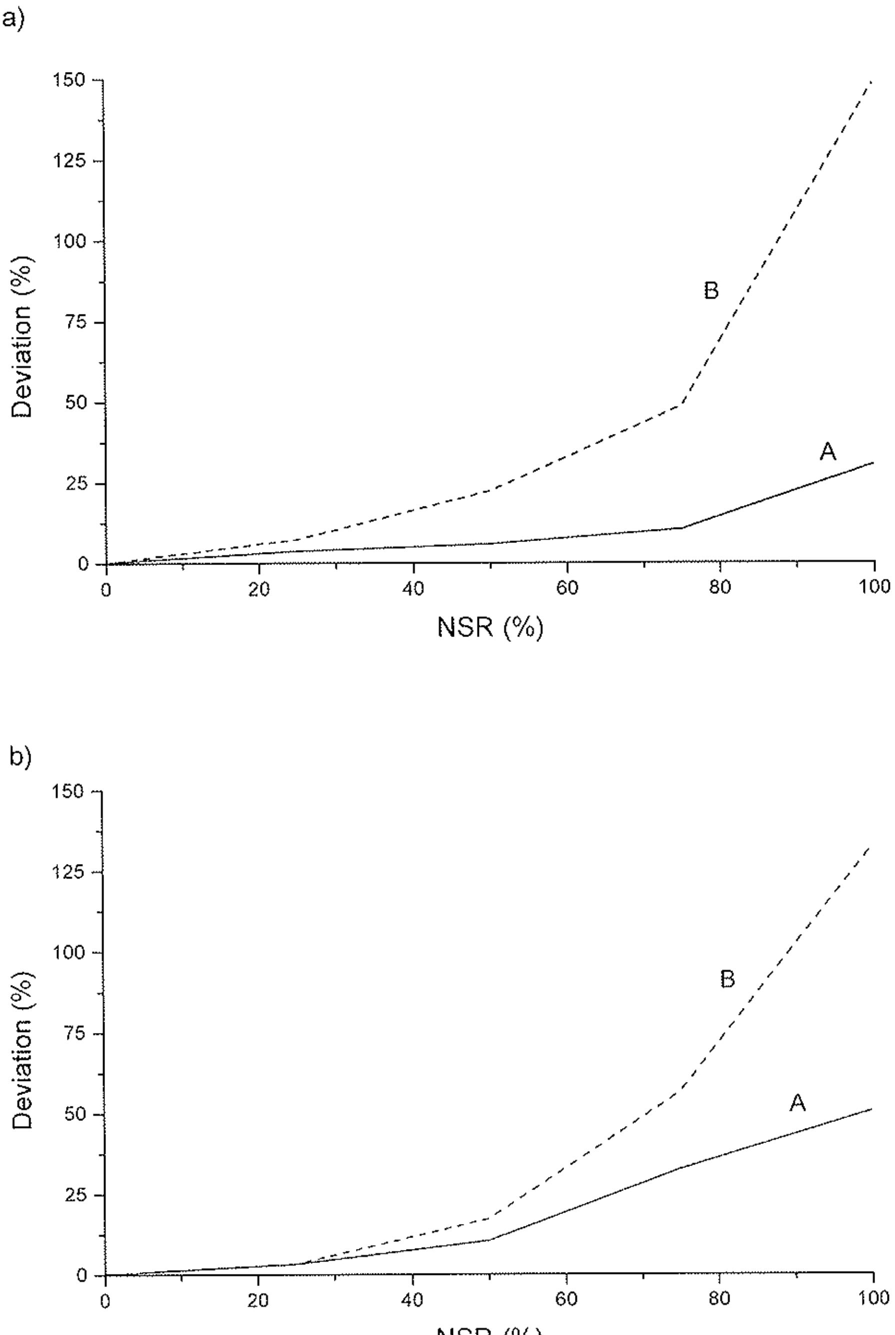

Fig. 3. Percentage deviations in the evaluation of Q without compensating for the random noise, corresponding to the cases Q = 50 (a) and Q = 100 (b). The curves correspond to the two different methods adopted in the evaluation of Q: the spectral-ratio method (A) and the frequency-shift method (B).

$$f_S = \int_0^{f_{max}} f\,[S(f) - A_N]df \Big/ \int_0^{f_{max}} [S(f) - A_N]df \ ,$$

$$\sigma_S = \int_0^{f_{max}} (f - f_S)^2[S(f) - A_N]df \Big/ \int_0^{f_{max}} [S(f) - A_N]df \ , \qquad (24)$$

where we neglect the contributions having $S(f) - A_N < 0$. Similarly, we redefined the centroid frequency f_R and variance σ_R of the receiver signal $R(f)$ as follows:

$$f_R = \int_0^{f_{max}} f\,[R(f) - A_N]df \Big/ \int_0^{f_{max}} [R(f) - A_N]df \ ,$$

$$\sigma_R = \int_0^{f_{max}} (f - f_R)^2[R(f) - A_N]df \Big/ \int_0^{f_{max}} [R(f) - A_N]df \ , \qquad (25)$$

where we neglect the contributions with $R(f) - A_N < 0$.

Fig. 4 shows the deviations corresponding to $Q = 50$ (a) and $Q = 100$ (b), calculated by using eqs. (24) and (25). We may conclude that for $Q \leq 50$ the two methods are nearly equivalent. For $Q > 50$ the two methods are equivalent only for NSR $\leq 50\%$, while for NSR $> 50\%$ the frequency-shift method is better.

2D simulations

In this example, we use a more realistic model, based on attenuation measurements in a homogeneous medium (Pierre shale), made by McDonal et al. (1958) near Limon, Colorado. They reported a constant-Q behaviour with attenuation $\alpha = 0.3177$ f (nepers/km). According to eq. (8), since c_0 is approximately 2133.3 m/s, the quality factor is $Q \cong 32.5$. We consider a reference frequency $f_0 = 250$ Hz, corresponding to the dominant frequency of the seismic source used in the experiments. The medium is discretized with uniform vertical and horizontal grid spacings of 2 m, and 77×77 grid points. The spatial derivatives are calculated with the Fourier method by using the fast Fourier transform (FFT) (Kosloff and Baysal, 1982). The source, applied at the centre of the mesh, is a Ricker-type wavelet described by eq. (11). The time step used in this simulation is 0.05 ms, and the receiver is located at 40 m from the source. With respect to the 1D simulation, we enlarged the source bandwidth and reduced the source-receiver (offset) distance. Absorbing strips

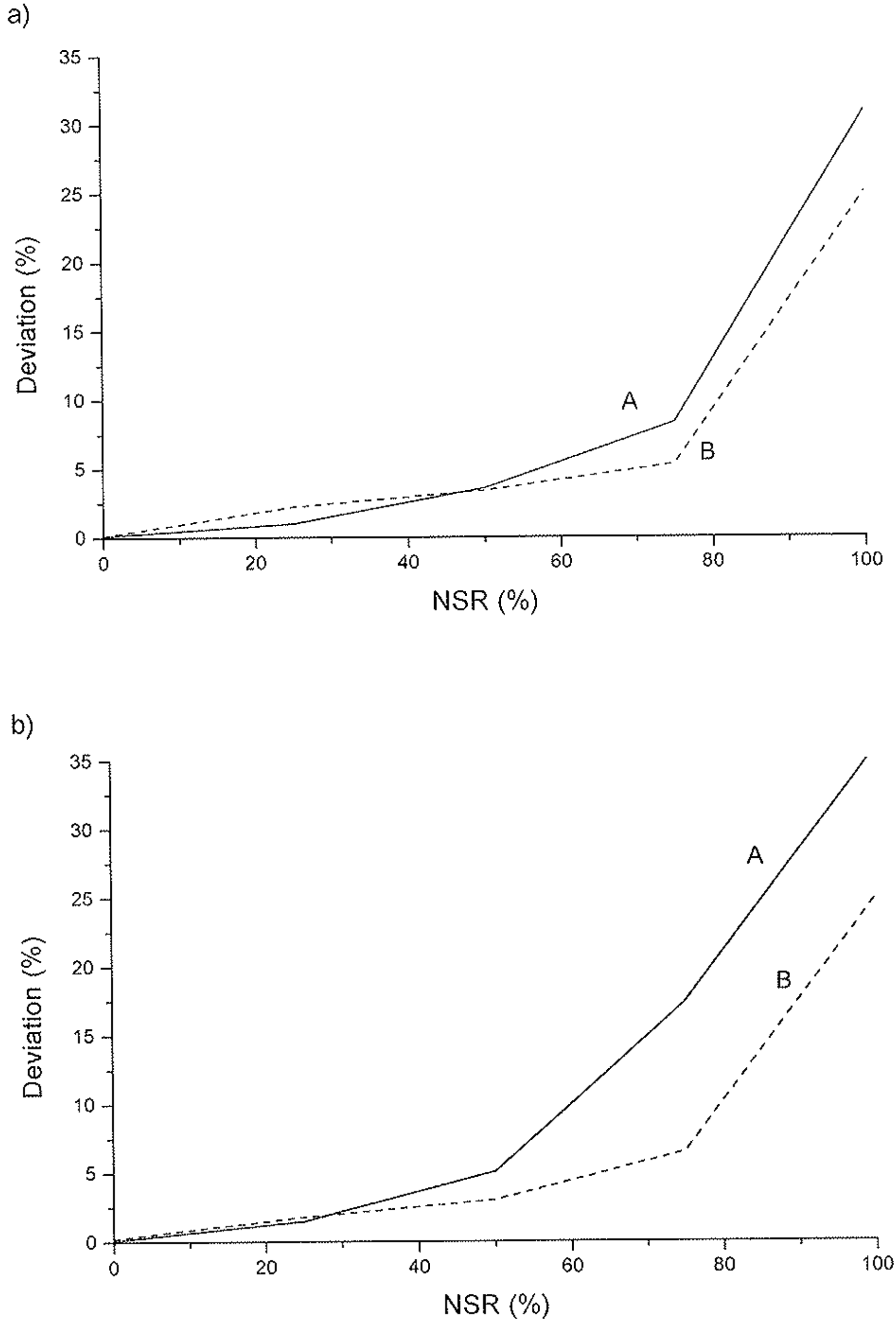

Fig. 4. Percentage deviations in the evaluation of Q after compensating for the random noise, corresponding to the cases Q = 50 (a) and Q = 100 (b). The curves correspond to the two different methods adopted in the evaluation of Q: the spectral-ratio method (A) and the frequency-shift method (B).

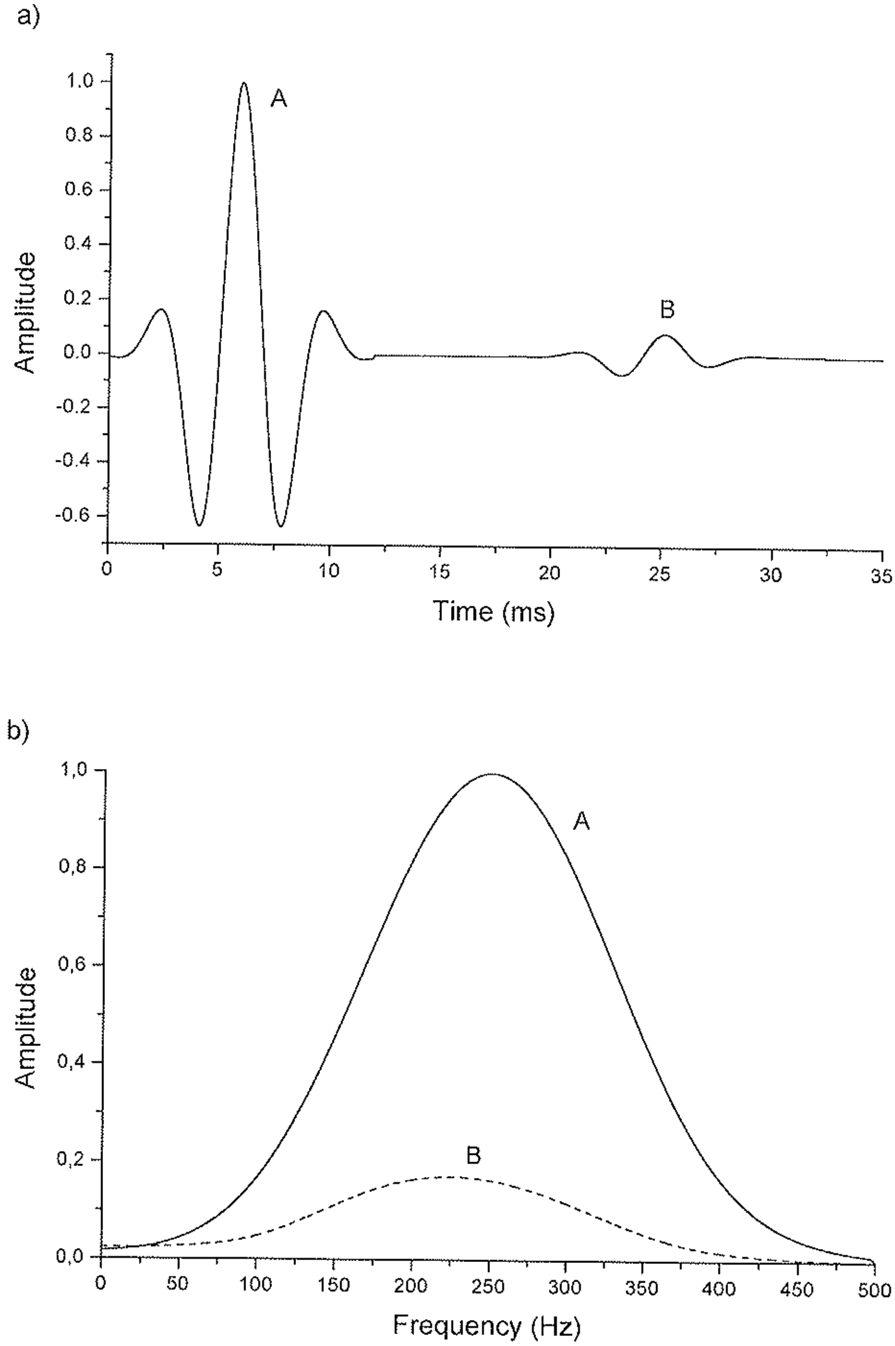

Fig. 5. Time propagation (a) and frequency spectra (b) corresponding to the 2D simulation (Pierre shale) without random noise, where A indicates the source and B the output signal. The reference frequency is $f_0 = \omega_0/(2\pi) = 250$ Hz, the offset is $x = 40$ m and the phase velocity corresponding to the dominant frequency is $c_0 = 2133.3$ m/s.

of width 12 grid points are implemented at the four boundaries of the mesh (Carcione et al., 1988). As shown by Carcione et al. (2002), the finite-difference algorithm is more accurate and efficient when the equation is expressed in the dilatation formulation, and the agreement between the numerical and analytical solution (based on the 2D Green's function) is excellent when the memory operator length is about 40. Fig. 5 shows the time propagation (a) and frequency spectra (b) of the numerical simulation without noise. Note that in this case the output signal is affected also by the geometrical-spreading attenuation. Fig. 6 compares two snapshots of the dilatation field computed at 20 ms, where (a) corresponds to the case without noise, and (b) to the case with noise (NSR = 10%). Fig 7(a) shows the errors in the computation of Q and we notice that the spectral-ratio method is slightly better for NSR $\leq$ 50%, while the frequency-shift method is better when NSR $>$ 50%.

Finally, we performed a 1D simulation using the Pierre shale parameters and an offset of 300 m. The resulting output signal has the same spectral peak amplitude obtained in the 2D simulation. Fig 7(b) shows the errors in the evaluation of Q: the two methods are practically equivalent, and we notice that the errors are quite lower if compared to the 2D simulation [Fig. 7(a)]. This means that the errors increase considerably with decreasing offsets, in agreement with the fact that the attenuation effects are generally observed with accuracy over large distances.

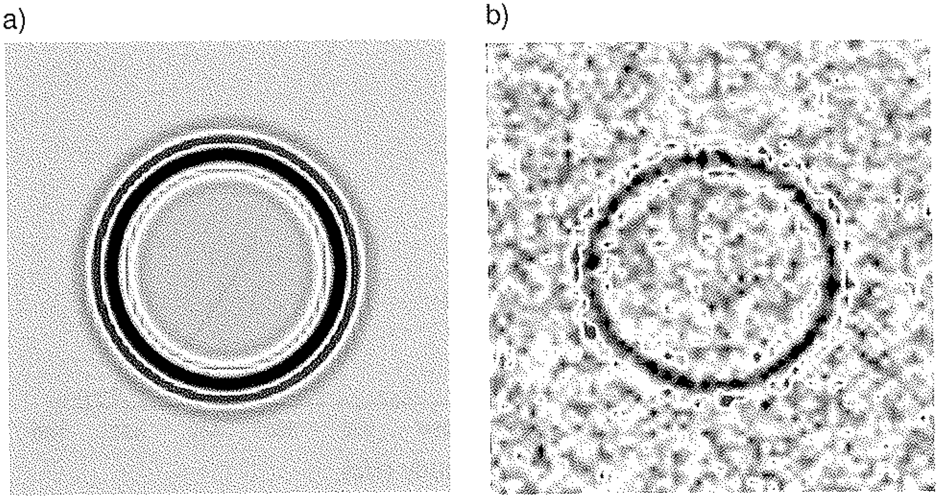

Fig. 6. Snapshots at 20 ms of the dilatation 2D field in a dissipative model of Pierre shale, without random noise (a) and with random noise (NSR = 10%) (b).

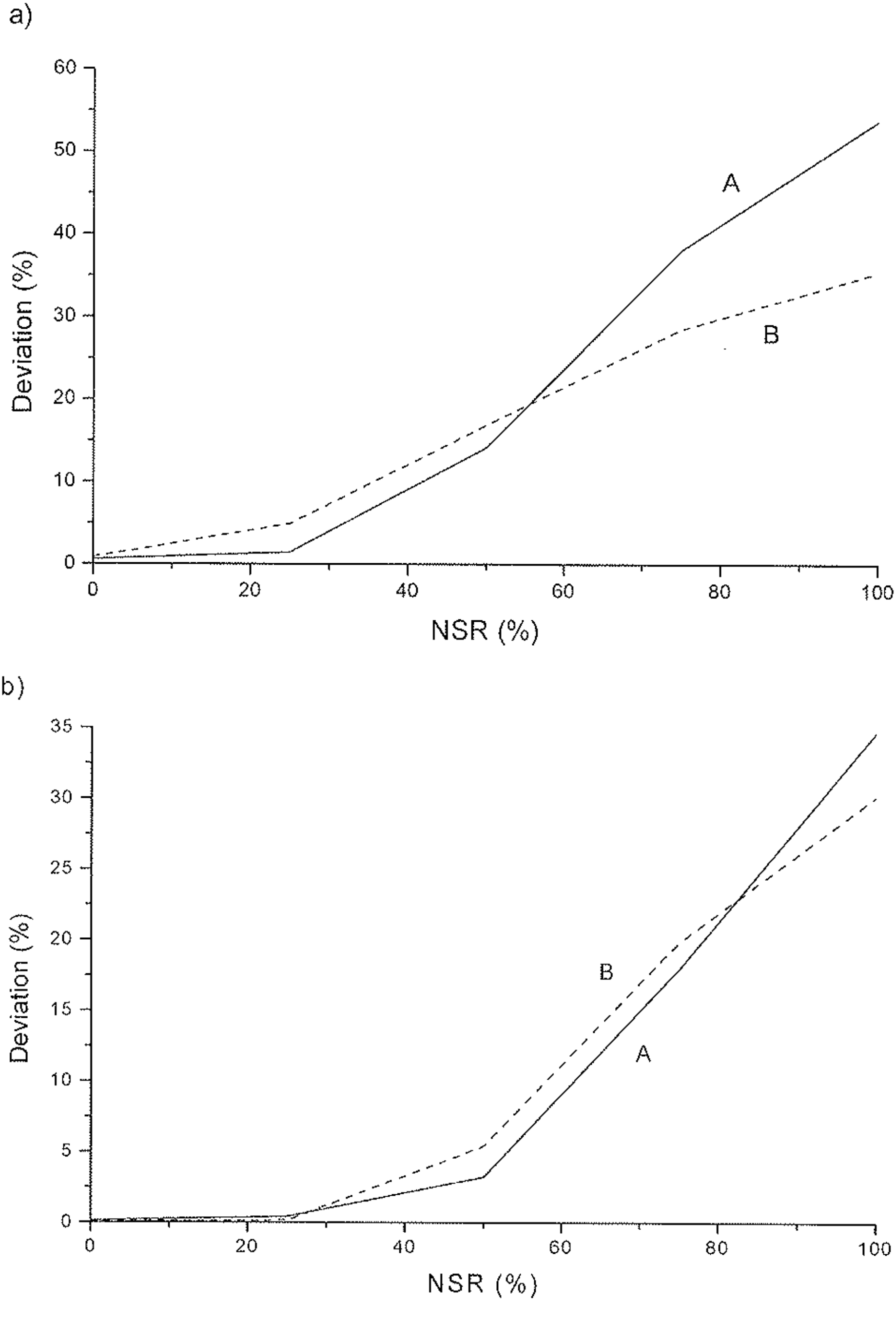

Fig. 7. Percentage deviations in the evaluation of Q after compensating for the random noise, corresponding to the 2D Pierre shale case (a) and 1D Pierre shale case with offset x = 300 m (b). The curves correspond to the two different methods adopted in the evaluation of Q: the spectral-ratio method (A) and the frequency-shift method (B).

CONCLUSIONS

We tested successfully the reliability of the spectral-ratio and frequency-shift methods for estimating the intrinsic quality factor Q in the presence of random noise. We simulated constant-Q wave propagation in a 1D and 2D (Pierre shale) homogeneous viscoacoustic media. For the 1D model we solved the equation analytically in the frequency domain using the Green's function, while for the 2D model we used a numerical algorithm based on fractional derivatives. Several tests were made to estimate the intrinsic quality factor, using in the simulations different Q values and random noise levels. The two methods are very accurate (deviations less than 0.3%) and equivalent when there is no noise, while in presence of random noise the spectral-ratio technique is better than the frequency-shift technique. However, when the magnitude of random noise is evaluated from the spectrum and compensated, the accuracy of the calculation is highly improved and for low values of noise levels the two methods are equivalent. Moreover, the frequency-shift method is better than the spectral-ratio method when the noise level is high. Finally, since over large distances the attenuation effects are more evident, we verified that the accuracy of the evaluation of Q using both methods increases with increasing offsets.

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