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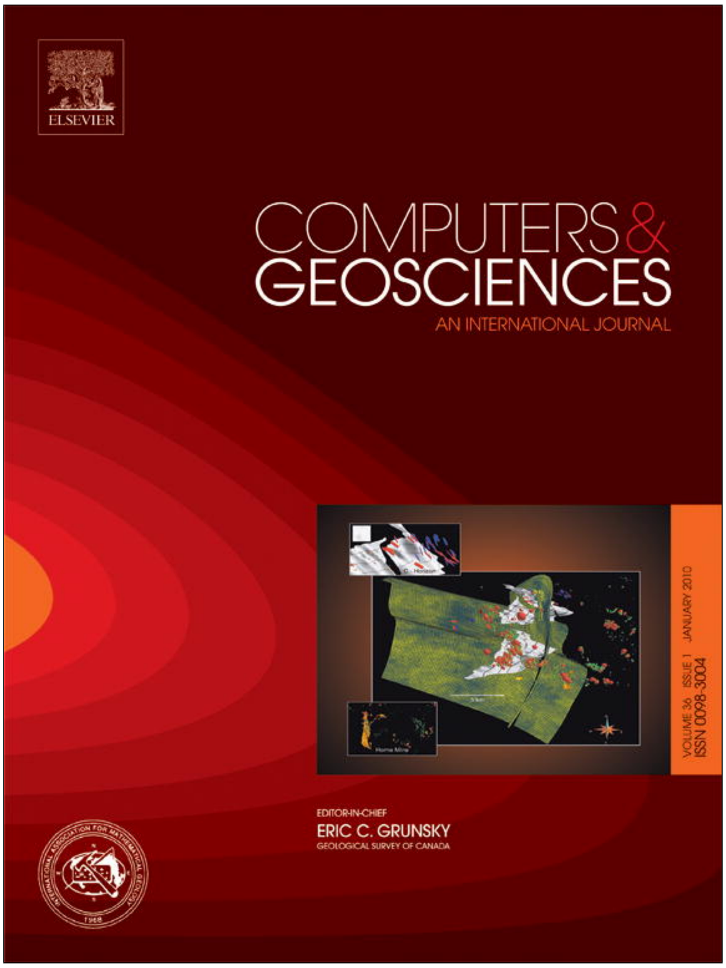

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A viscoelastic representation of wave attenuation in porous media

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ABSTRACT

The theories developed by White and co-workers describe the complex moduli of a medium partially saturated with spherical gas pockets and those of stratified layers composed of two heterogeneous porous media. A generalization to gas patches of arbitrary shape has been given by Johnson. These models represent the mesoscopic-loss mechanism, which is one of the most significant causes of attenuation of seismic waves in reservoir rocks. Comparison of White's and Johnson's models show that, as the patch shape complexity increases, the patch geometry affects much more the relaxation frequency than it affects the maximum loss. The simulation of synthetic seismograms requires solving Biot's differential equations with very small grid spacings, because the loss mechanism involves the conversion of fast P-wave energy to diffusion energy in the form of the Biot slow wave. Because the wavelength of this wave can be very small, the poroelastic solution requires a very large amount of storage and computer time. An efficient approach is to approximate White's moduli by the Zener model and then solve the single-phase viscoelastic differential equations.

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1. Introduction

The study of the effects of saturation and fluid type on wave propagation, particularly in the case of reservoir rocks, requires the modeling of rocks as porous media. Recent studies (e.g., Pride et al., 2004) have shown that the major cause of attenuation and velocity dispersion at seismic frequencies in porous media is wave-induced fluid flow at the mesoscopic scale, larger than the pore size but smaller than the wavelength (typically tens of centimeters). White (1975) and White et al. (1975) were the first to introduce the mesoscopic-loss mechanism based on approximations in the framework of Biot theory of poroelasticity (Biot, 1956a, b; Biot, 1962). Biot's theory, originally formulated for a single-fluid system, was further refined and generalized for two-fluid systems by many authors (Brutsaert, 1964; Berryman et al., 1988; Santos et al., 1990a, b; Tuncay and Corapcioglu, 1996; Lo et al., 2005).

White's models predict the dependence of wave velocity and attenuation as a function of frequency and dimension of the mesoscopic-scale heterogeneities of the medium. The model given in White et al. (1975) considers a periodically stratified medium and describes the amount of attenuation (and velocity dispersion) caused by different types of heterogeneities in the rock properties, namely, porosity, grain and frame moduli, permeability and fluid properties. In this case, the complex modulus obtained with this theory is the one perpendicular to the layering plane. Carcione and Picotti (2006) find that the most significant loss mechanisms are a result of porosity variations and partial saturation, where one of the fluids is very stiff and the other is very compliant. On the other hand, White (1975) describes a medium partially saturated with water and spherical gas pockets. In both models, the fluid effects on wave velocity and attenuation depend on the frequency range. At low frequencies, the fluid has enough time to achieve pressure equilibration (relaxed regime), and Gassmann's modulus properly describes the saturated bulk modulus. At high frequencies, the fluid cannot relax, and this state of unrelaxation induces pore-pressure gradients. Consequently, the bulk and shear moduli have larger values than at low frequencies.

Attenuation and velocity dispersion are caused by fluid flow between patches of different pore pressures. The critical fluid-diffusion relaxation length is proportional to the square root of the ratio of permeability to frequency. At seismic frequencies the relaxation length is very large, and the pressure is nearly uniform throughout the medium. As frequency increases, differences in pore-pressure can cause a significant increase in P-wave velocity. The theory, based on Biot's theory of poroelasticity, predicts that increasing fluid viscosity or decreasing permeability shifts the relaxation peaks towards lower frequencies. Dutta and Seriff (1979) solved the problem exactly by using Biot theory and confirmed the accuracy of White's results. They point out a mistake in White (1975), where White uses the P-wave modulus instead of the bulk modulus to derive the complex bulk modulus.

The situation in real porous material is obviously more complex. In Johnson (2001) and Müller and Gurevich (2005) a

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generalization of the White periodic model for patches of arbitrary shape is given. Besides the usual parameters of White theory, the model developed by Johnson (2001) has two other geometric parameters: the specific surface area S/V and T. The parameter S/V depends on the shape of the patches, whereas the parameter T is related to the mean size of the patches. Müller and Gurevich (2005) developed a theory for elastic wave propagation in a fluid-saturated porous medium with a random distribution of 3-D inhomogeneities. Applying the method of statistical smoothing to Biot's equations of poroelasticity, they derived an explicit expression for the effective wave number of a P-wave that accounts for the effect of wave-induced flow. These models show that, in a complex 3-D space, the observed frequency dependency of attenuation due to fluid flow is independent of the type of disorder (periodic or random). In other words, the shape of the attenuation and dispersion curves are those typical of a relaxation mechanism. Moreover, if ω is the angular frequency, the predicted dissipation factor $1/Q$ for the random model is proportional to ω and $1/\sqrt{\omega}$ at the low- and high-frequency limits, respectively. These asymptotes coincide with those predicted by the periodicity-based approaches, so the qualitative aspects of the physics obtained using these simple models have a quite general validity.

It is very common, in reservoir problems, that the gas is distributed in patches whose sizes are in the mesoscopic scale. Cadoret et al. (1995) investigated the phenomenon in the laboratory at the frequency range 1–500 kHz. Two different saturation methods result in different fluid distributions and yield two different values of velocity for the same saturation. Imbibition by depressurization produces a very homogeneous saturation, whereas drainage by drying produces heterogeneous saturations at high water saturation levels. In the latter case, the experiments show considerably higher velocities, as predicted by White's model.

Picotti et al. (2007) solved the Biot equations of motion for a two-fluid system and showed that the results are in good agreement with the theoretical values predicted by the White theory. However, the computation of synthetic seismograms in the presence of mesoscopic heterogeneities requires solving Biot's differential equations with very small grid spacings (Picotti et al., 2007). Because this solution implies a very large computational effort, we approximate White's relaxation peaks by the Zener model (Carcione, 2007) in order to reduce the problem to the solution of equivalent single-phase viscoelastic differential equations.

2. The dynamic bulk modulus

The dynamic bulk modulus $\tilde{K}(\omega)$ describes the crossover from Gassmann–Wood's (GW) result at low frequencies to the Gassmann–Hill (GH) result at high frequencies. When the pore space is partially saturated with two very different fluids, such as gas and water, a fast P-wave traveling in the medium induces very different pore pressures in the two regions, which tend to equilibrate. This process is governed by the so-called Biot acoustic slow wave, a diffusive phenomenon in which the diffusivity is given by

$$D(K_{fj}) = \left(\frac{\kappa}{\eta\phi^2}\right)\frac{PR-Q^2}{P+2Q+R}, \quad j=1,2 \tag{1}$$

(Johnson, 2001; Carcione, 2007), where K_{fj} is the bulk modulus of the j-th fluid that diffuses away from the interface separating the two regions. Omitting the subindex j for clarity the explicit expressions for P, Q and R are given by

$$P = M(\alpha-\phi)^2 + K_m + \tfrac{4}{3}\mu_m,$$

$$Q = \phi M(\alpha-\phi),$$

$$R = M\phi^2 \tag{2}$$

(Biot and Willis, 1957; Carcione, 2007), where K_m is the dry-rock bulk modulus and μ_m is the dry-rock shear modulus. The parameter α (also known as the Biot–Willis coefficient) and M are given by

$$\alpha = 1 - \frac{K_m}{K_s},$$

$$M(K_f) = \left(\frac{\alpha-\phi}{K_s} + \frac{\phi}{K_f}\right)^{-1}, \tag{3}$$

where K_s is the solid-grain bulk modulus. The effective P-wave bulk modulus of the two regions is

$$K_E = \frac{E_m M}{E_G} \tag{4}$$

(Carcione and Picotti, 2006), where

$$E_m = K_m + \tfrac{4}{3}\mu_m \tag{5}$$

is the dry-rock P-wave modulus, and

$$E_G = K_G + \tfrac{4}{3}\mu_m. \tag{6}$$

The Gassmann bulk modulus K_G is given by

$$K_G = K_m + \alpha^2 M. \tag{7}$$

As shown by White (1975), slow-wave diffusion induces wave-velocity dispersion and attenuation of the fast P-wave, which depends mostly on the size of the gas pockets (saturation), frequency, permeability and porosity of the rocks. At very low frequencies, there is enough time for pore pressure to equilibrate to a constant value. Therefore, the fluid pressure is uniform (isostress state), and the effective modulus of the pore fluid is given by Wood's (1955) modulus, K_W, which is exact for the static modulus of two fluids:

$$K_W = \left(\frac{S_1}{K_{f1}} + \frac{S_2}{K_{f2}}\right)^{-1}, \tag{8}$$

where $S_j,\ j=1,2$, is the saturation of the j-th fluid. In this case, the effective bulk modulus of the composite at the low frequency limit is given by the Gassmann expression

$$K_{GW} = K_m + \alpha^2 M(K_W) \tag{9}$$

(Johnson, 2001; Dutta and Odé, 1979), and it is independent of the spatial distribution of the fluids. The process of equilibration is governed by the diffusion equation whose diffusivity constant is given by (1). After some algebra, Eq. (1) may be rewritten in a more simple form:

$$D(K_f) = \frac{\kappa K_E}{\eta} \tag{10}$$

and the critical fluid diffusion relaxation length is given by

$$L_c = \sqrt{D/\omega}. \tag{11}$$

On the other hand, when the frequency is sufficiently high (e.g. smaller diffusion lengths) the pore pressures in the two phases do not have enough time to equilibrate within one half cycle. Consequently, the pressure is not uniform, but, it can be assumed to be constant within each phase. In such a situation, the fluid flow effects can be ignored and Hill's theorem (Hill, 1964; Norris, 1993) gives the composite bulk modulus at the high-frequency limit:

$$K_{GH} = \left(\frac{S_1}{E_{G1}} + \frac{S_2}{E_{G2}}\right)^{-1} - \frac{4}{3}\mu_m. \tag{12}$$

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The high-frequency P-wave modulus is given by

$$E_\infty = K_{GH} + \tfrac{4}{3}\mu_m. \tag{13}$$

3. Models and wave properties

3.1. White's and Johnson's models

To illustrate the frequency dependence of the crossover phenomenon we consider White's and Johnson's approaches with a regular distribution of patches, which are more extensively outlined in the appendices. White (1975) and White et al. (1975) were the first to introduce the mesoscopic-loss mechanism based on approximations in the framework of Biot theory. They considered a periodic ensemble of gas pockets in a water-saturated porous medium. Their first model consisted of porous layers alternately saturated with water and gas, respectively. Then they proposed a model consisting of a periodic ensemble of water-saturated cubic cells each containing a sphere saturated with gas in which the fluid flow occurs at the sphere's interface. For simplicity in the calculations, White considered an outer sphere instead of a cube. Thus, the system consists of two concentric spheres, where the volume of the outer sphere is the same as the volume of the original cube. Johnson (2001) developed a generalization of the White model for periodic ensembles of fluid patches of arbitrary identical shape. In addition to the usual parameters of White theory, this model has two other geometric parameters: the specific surface area S/V and T. The parameter S/V depends on the shape of the patches, whereas the parameter T is governed by the mean size of the patches (Johnson, 2001). For these reasons, we refer to S/V as the *shape factor* and to T as the *size factor*. For a given shape, Johnson (2001) derived asymptotic solutions for low and high frequencies. The solution for intermediate frequencies was proposed using the simplest function that ensured causality of the solution.

3.2. Phase velocity and quality factor

Once we have determined $\tilde{K}(\omega)$, the complex P-wave velocity is given by

$$v(\omega) = \sqrt{\frac{E(\omega)}{\rho}}, \tag{14}$$

where $E(\omega)$ is the complex P-wave modulus

$$E(\omega) = \tilde{K}(\omega) + \tfrac{4}{3}\mu_m. \tag{15}$$

The dry-rock shear modulus, μ_m is assumed frequency-independent and everywhere constant. Moreover, $\rho = (1-\phi)\rho_s + \phi\rho_f$ is the bulk density, ρ_s is the grain density, and ϕ is the porosity. The density of the fluid mixture is

$$\rho_f = S_1\rho_{f1} + S_2\rho_{f2}, \quad S_2 = 1 - S_1, \tag{16}$$

where ρ_{f1} and ρ_{f2} are the densities of fluids 1 and 2 (gas and water in White's theory). The complex bulk modulus (15) can be expressed as $E = |E|\exp(i\theta)$, where θ is the loss angle. We use the concept of complex velocity to obtain the phase velocity and loss angle as a function of angular frequency ω. They are simply given by

$$v_p = \left[\mathrm{Re}\left(\frac{1}{v}\right)\right]^{-1} \tag{17}$$

and

$$\theta = \tan^{-1}\left[\frac{\mathrm{Im}(v^2)}{\mathrm{Re}(v^2)}\right] = \tan^{-1}\left(\frac{1}{Q}\right), \tag{18}$$

where Re and Im denote real and imaginary parts, respectively. Then, the relation between the loss angle and the standard definition of quality factor in viscoelasticity is $Q^{-1} = \tan\theta$.

3.3. The Zener model

To obtain the phase velocity and quality factor of the equivalent viscoelastic medium, we use the Zener model (Carcione, 2007). The dimensionless complex modulus of a Zener element can be expressed as

$$C(\omega) = \left(\frac{\tau_\sigma}{\tau_\varepsilon}\right)\left[\frac{1+(p\tau_\varepsilon)}{1+(p\tau_\sigma)}\right], \tag{19}$$

where $p = i\omega$ is the imaginary frequency and τ_σ and τ_ε are relaxation times. The quality factor associated with C is equal to $\mathrm{Re}(C)/\mathrm{Im}(C)$. Its minimum value is located at

$$\omega_0 = \frac{1}{\sqrt{\tau_\sigma\tau_\varepsilon}} \tag{20}$$

and is equal to

$$Q_0 = \frac{2x}{x^2-1}, \quad x = \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)^{1/2}, \tag{21}$$

where $f_0 = \omega_0/(2\pi)$ is the central frequency of the relaxation peak, and $1/Q_0$ is the maximum dissipation factor (e.g., Carcione, 2007). Using ω_0 and Q_0 as parameters, we have

$$\tau_\varepsilon = \frac{x}{\omega_0}, \quad \tau_\sigma = \frac{1}{x\omega_0}, \tag{22}$$

where x is a solution of Eq. (21):

$$x = \frac{1+\sqrt{1+Q_0^2}}{Q_0}. \tag{23}$$

The complex velocity v is the key quantity to obtain the phase velocity and quality factor of the equivalent viscoelastic medium. Assuming that there is no shear relaxation, the complex P-wave modulus is

$$E = (E_\infty - \tfrac{4}{3}\mu_m)C(\omega) + \tfrac{4}{3}\mu_m, \tag{24}$$

where ω is the angular frequency and E_∞ is the high-frequency P-wave modulus given by (13). If μ_m is not constant, it is necessary to set $E = E_\infty C(\omega)$.

4. Matrix and fluid properties

We consider a uniform porosity $\phi = 0.3$ and permeability $\kappa = 1$ D, which are typical values for reservoir rocks (Bear, 1972). Permeability is related to porosity by the Kozeny–Carman relation

$$\kappa = \frac{B\phi^3 D^2}{(1-\phi)^2} \tag{25}$$

(Mavko et al., 1998), where D is the grain diameter ($D = 80\,\mu$m for a sandstone), and $B = 0.003$ is a geometric factor. We use the model of Krief et al. (1990) to obtain the dry-rock moduli K_m and μ_m. The porosity dependence is consistent with the concept of critical porosity because the moduli should be small above a certain value of the porosity (usually from 0.4 to 0.6). The moduli are given by

$$K_m = K_s(1-\phi)^{3/(1-\phi)},$$
$$\mu_m = K_m\mu_s/K_s, \tag{26}$$

where K_s and μ_s are the bulk and shear moduli of the solid grains. The mesoscopic-loss effect is enhanced when the stiffnesses of the two fluids in the pore space are quite different, such as gas and

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Table 1
Material properties.

Matrix	
Grain bulk modulus, K_s	37 GPa
Grain shear modulus, μ_s	44 GPa
Grain density, ρ_s	2650 kg/m^3
Dry – rock bulk modulus, K_m	8 GPa
Dry – rock shear modulus, μ_m	9.5 GPa
Porosity, ϕ	0.3
Permeability, κ	1 D
Water	
Bulk modulus, K_f	2.25 GPa
Density, ρ_f	1040 kg/m^3
Viscosity, η	3 cP
Gas	
Bulk modulus, K_f	0.012 GPa
Density, ρ_f	78 kg/m^3
Viscosity, η	0.15 cP

water. The properties of the fluids, which correspond to those of water and methane (CH_4) at 1-km depth given in Carcione and Picotti (2006), depend on temperature and pressure, which in turn depend on depth z (Friedman, 1963; Morse and Ingard, 1986). The material properties of matrix and fluids are given in Table 1.

5. Examples

In order to compare the White and Johnson models, we consider the following two simple geometries of the patches: (a) the concentric spheres geometry wherein a gas-saturated sphere of radius a is surrounded by an outer sphere of radius b, and (b) the plane layer geometry in which a gas-saturated layer of thickness d_1 and a water-saturated layer of thickness d_2 are periodically repeated as in White et al. (1975). Thus, we have a total of four cases: White layered, White spherical, Johnson layered and Johnson spherical. We compare patches of the same

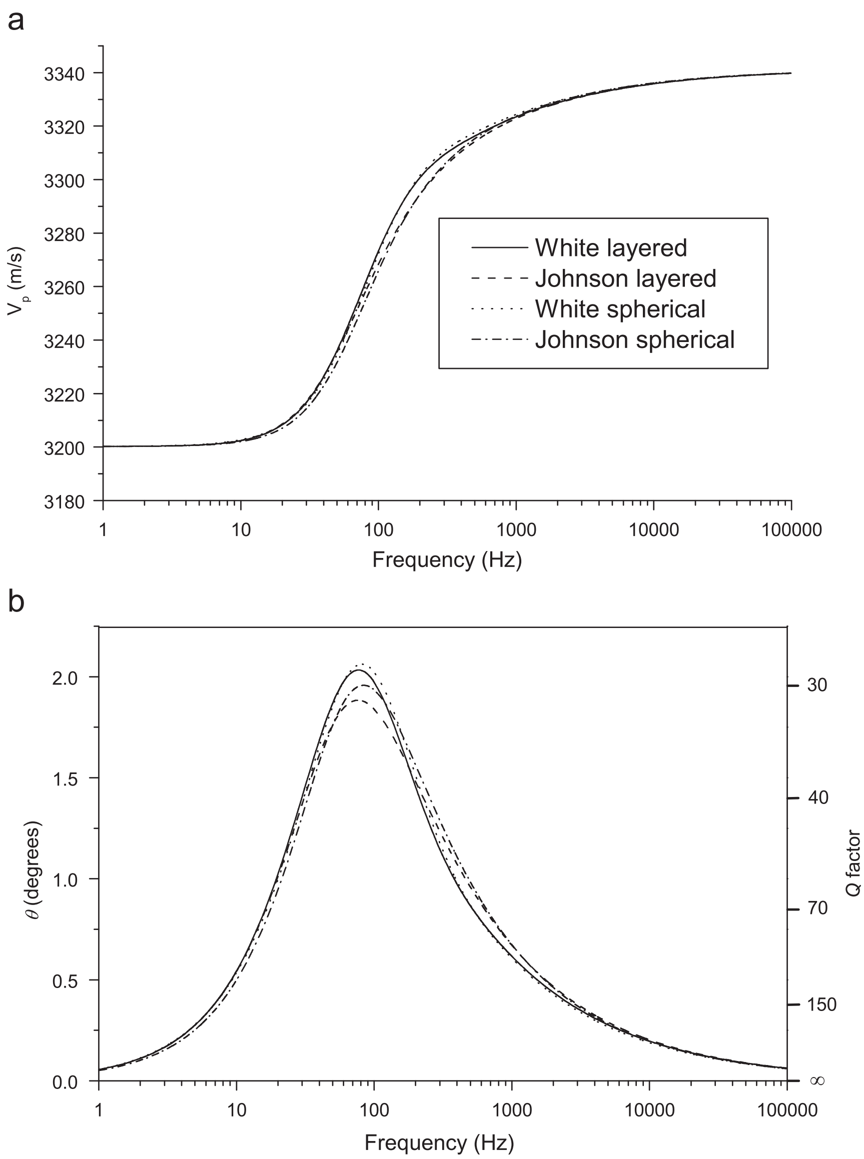

Fig. 1. Phase velocity (a) and loss angle (and quality factor) (b) curves corresponding to four cases: White layered, White spherical, Johnson layered and Johnson spherical. Material properties are shown in Table 1 and gas saturation is 50%. Period of the layered model $L = d_1 + d_2$ is set equal to outer radius of spherical model: $L = b = 40$ cm. Models are in good agreement.

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size setting the period of the layered model $L = d_1 + d_2$ equal to the outer radius of the spherical model: $L = b = 40$ cm.

Fig. 1 shows the corresponding phase velocities (a) and loss angles (b) of the four cases, all at the gas saturation $S_2 = 50\%$. It is immediately apparent that, even if the geometry of the patches are completely different, the shapes of both the velocity and loss angle curves are quite similar in all the four cases. Also, the values of the crossover frequency and maximum loss (i.e., the position of the attenuation peak) are very close to each other. This result is in agreement with the fact that the shape and size factors have almost the same values for the two geometries: $S/V = 5.0$, 4.72 and $1000T = 0.397, 0.363$ for the layered and spherical geometries, respectively.

In order to evaluate the sensitivity of the model to a change in the mobility of the fluid, we computed the curves using a permeability of 300 mD. In this case, the porosity is not related to permeability by Eq. (25), and it is the same of the previous case. Permeability and viscosity describe the mobility of the fluid in the pore space. As shown in Carcione and Picotti (2006), a decrease in permeability (or increase in viscosity) decreases mobility. The relaxation peak moves towards low frequencies for decreasing permeability, in agreement with Eq. (A.5). For a permeability of 300 mD, the relaxation peak is downshifted to a frequency of about 23 Hz, while the maximum loss does not change.

Fig. 2 shows the corresponding phase velocities (a) and loss angles (b) for a gas saturation $S_2 = 10\%$. With respect to Fig. 1, we note that the shapes of both the velocity and loss angle curves are still similar in all the four cases, but the position of the attenuation peak is different for the two geometries. In particular, the relaxation frequency is lower and the maximum loss is slightly higher for the spherical geometry. This result reflects the fact that the shape and size factors have different values: $S/V = 5.0, 1.62$ and $1000T = 0.198, 0.629$ for the layered and spherical geometries, respectively. For both geometries, the White and Johnson models are in good agreement. This downshift of the relaxation frequency is in agreement with Eqs. (A.5) and (B.4). Comparing the two equations, we find that the relaxation frequency is the same for the two geometries when $b - a \approx d_2/(2\sqrt{2})$. This condition is satisfied for $S_2 = 50\%$, whereas for $S_2 = 10\%$ we have $b - a \gg d_2/(2\sqrt{2})$.

In the previous two examples we considered two geometries of the patches, varying only the saturation of gas and water. Let us

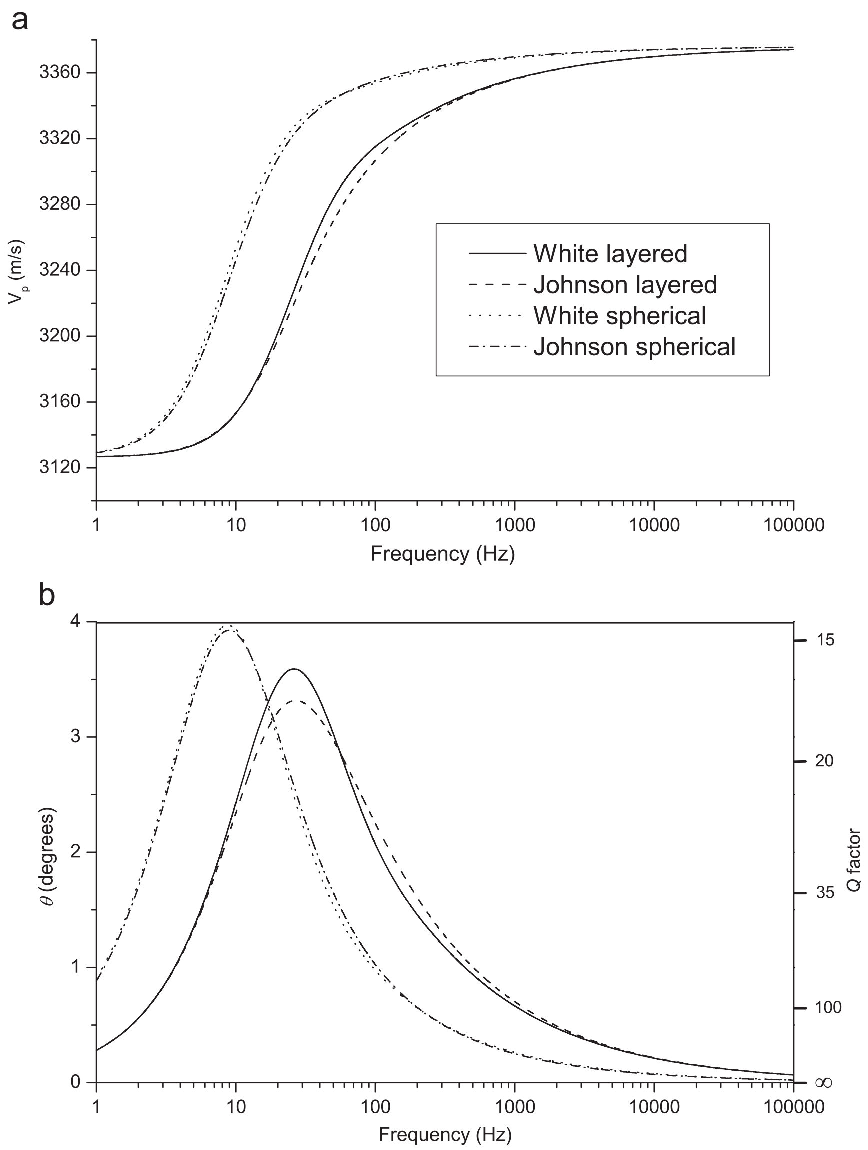

Fig. 2. Phase velocity (a) and loss angle (and quality factor) (b) for a gas saturation of 10%.

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keep the saturation and the size factor constant, in order to examine how the shape of the patches affects the attenuation curves. We start from a spherical patch of radius $b = 40$ cm, saturation $S_2 = 50\%$, size factor $1000T = 0.363$ and shape factor $S/V = 4.72$. Then we deform gradually the patch from a sphere to a fractally rough shape, by simply changing only the shape factor. The values considered are the following: $S/V = 9.44, 23.6$ and 47.2, which correspond, respectively, to twice, five times and ten times the initial spherical shape factor. Fig. 3 shows the phase velocities (a) and loss angles (b) corresponding to the four values considered. We note that the shapes of the curves are similar, but when S/V increases (i.e., the irregularity of the shape increases), the relaxation peak moves towards higher frequencies, whereas the maximum loss decreases. In particular, starting from the sphere, we observe that if S/V doubles, both the relaxation frequency and the maximum loss change by the same percentage (about 20%). However, when S/V increases from twice to five times the initial value, the maximum loss decreases by 20%, while the relaxation frequency increases by 500%. Finally, when S/V increases from five times to ten times the initial value, the maximum loss decreases only by 8%, while the relaxation frequency increases by 400%. In other words, when the shape factor becomes twice the initial value for a sphere (regular shapes), the relaxation frequency and the maximum loss are affected in the same way. When the shape factor becomes five times and ten times the initial value (very irregular shapes) the patch geometry affects the relaxation frequency much more than it affects the maximum loss.

With the purpose of obtaining the parameters of an equivalent viscoelastic medium, we fit both the phase velocity and loss angle curves relative to the White layered case of Fig. 1, using the Zener model. The result is shown in Fig. 4, where the best fit is obtained with the parameters $f_0 = 77.1$ Hz and $Q_0 = 11.8$. In the same figure we also show the Zener best fit of both the phase velocity and loss angle curves relative to the White spherical case of Fig. 2, obtained with the parameters $f_0 = 8$ Hz and $Q_0 = 6.3$. We note that the Zener model fits, in both cases, better the loss-angle curve than the velocity curve. Moreover, the loss-angle curve is more reliable at seismic frequencies ($f < 250$ Hz) in the first case, and in the range 1–50 Hz in the second case. This is because the Zener model

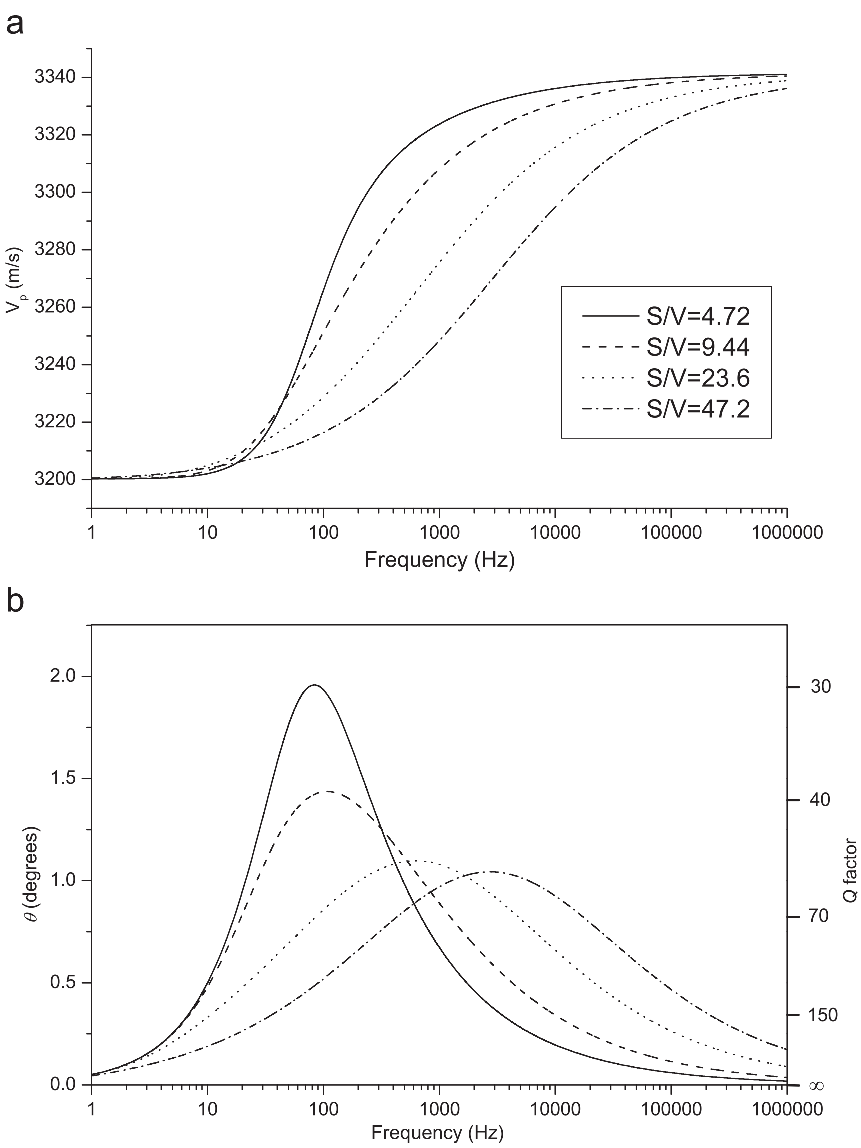

Fig. 3. Phase velocity (a) and loss angle (and quality factor) (b) corresponding to patches of arbitrary shape (Johnson's model) and a gas saturation of 50%. These curves correspond to the following shape factor values: $S/V = 4.72, 9.44, 23.6.$ and 47.2.

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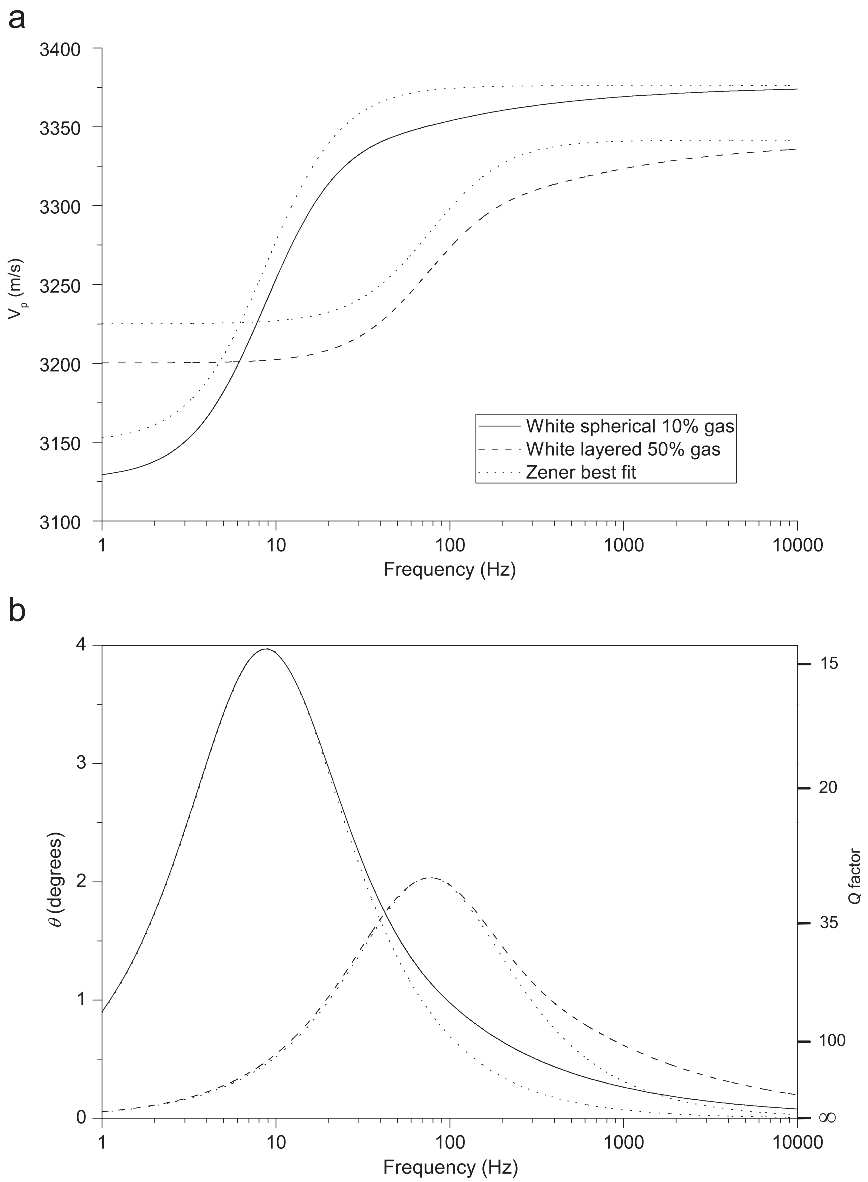

Fig. 4. Best fit of both phase velocity (a) and loss angle (and quality factor) (b) curves relative to White layered case of Fig. 1 and White spherical case of Fig. 2 using Zener model. Parameters of Zener model are $f_0 = 77.1$ Hz, $Q_0 = 11.8$ in first case, and $f_0 = 8$ Hz, $Q_0 = 6.3$ in the second.

is only an approximation of the mesoscopic-loss effect. The Zener model provides enough reliable approximations also for irregular geometries.

6. Conclusions

One of the major causes of wave attenuation in heterogeneous porous media is wave-induced flow of pore fluid at mesoscopic scales. In complex real media the behavior of attenuation as a function of frequency depends on the distribution and shape of inhomogeneities. We compared White's and Johnson's models in order to study how the shape of the patches influences the attenuation curves. The two models are in good agreement in the layered and spherical cases at a given saturation, and the geometry appears to affect only the position of the attenuation peak. Considering patches of arbitrary shape, the patch geometry affects both the relaxation frequency and the maximum loss. For regular geometries they are affected in the same way, whereas for fractally rough geometries, the former is much more affected than the latter.

The mesoscopic models can be effectively approximated by using the Zener viscoelastic model. The Zener model provides accurate fits when the patches have a regular geometry. On the other hand, despite that the fit is not mathematically perfect, the Zener model provides a good approximation also for irregular geometries. For practical purposes, the approximation is good enough to obtain synthetic seismograms in heterogeneous media. The advantages are that the use of very small grid spacings due to the presence of the Biot slow wave can be avoided. This implies that one can use a coarse grid, saving computer storage, particularly in three dimensions. Moreover, the viscoelastic modeling algorithm uses fewer material properties and field variables than the corresponding poroelastic modeling. Hence further storage saving and reduction of computer time can be achieved.

This research is of particular relevance for the simulation of wave propagation in reservoir rocks.

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Appendix A. White's model of a layered porous medium

Let us consider a periodic layered system composed of porous media 1 and 2 with thickness d_l, period d_1+d_2 and saturation $S_l=d_l/(d_1+d_2)$, $l=1,2$. White et al. (1975) obtained the complex bulk modulus for a P wave traveling along the direction perpendicular to the stratification. It is given by

$$E(\omega)=\left[\frac{1}{E_\infty}+\frac{2(r_2-r_1)^2}{\mathrm{i}\omega(d_1+d_2)(I_1+I_2)}\right]^{-1}. \tag{A.1}$$

Omitting the subindex l for clarity, we have for each medium:

$$r=\frac{\alpha M}{E_G}. \tag{A.2}$$

This is the ratio of fast P-wave fluid tension to total normal stress. Moreover,

$$I=\frac{\eta}{\kappa k}\coth\left(\frac{kd}{2}\right) \tag{A.3}$$

is an impedance related to the slow P-wave, where η is the fluid viscosity, κ is the permeability, and

$$k=\sqrt{\frac{\mathrm{i}\omega\eta}{\kappa K_E}} \tag{A.4}$$

is the complex wavenumber of the slow P-wave. The parameters E_∞, M, K_E and E_G are given by Eqs. (13), (3), (4) and (6), respectively. In this model the fluid pressures will equilibrate if the Biot slow-wave relaxation length L_c, given by (11), is comparable to the period of the stratification. The approximate transition frequency separating the relaxed and unrelaxed states (i.e., the approximate location of the relaxation peak) is

$$\omega_c=\frac{16\kappa K_{E2}}{\eta_2 d_2^2} \tag{A.5}$$

(Dutta and Seriff, 1979; Carcione et al., 2003), where the subindex 2 refers to water for a layered medium alternately saturated with water and gas. At this reference frequency, the critical length L_c equals the mean layer thickness or characteristic length of the inhomogeneities (Gurevich and Lopatnikov, 1995). Eq. (A.5) indicates that the mesoscopic-loss mechanism moves towards the low frequencies with increasing viscosity and decreasing permeability.

Appendix B. White's model of spherical gas pockets

White (1975) assumed spherical patches much larger than the grains but much smaller than the wavelength. He developed the theory for a gas-filled sphere of porous medium of radius a located inside a water-filled sphere of porous medium of outer radius b ($a<b$). Let us denote the saturation of gas and water by S_1 and S_2, respectively. Then

$$S_1=\frac{a^3}{b^3},\quad S_2=1-S_1. \tag{B.1}$$

Assuming that the dry-rock, grain moduly and permeability of the different regions are the same, the dynamic bulk modulus as a function of frequency is given by

$$\tilde{K}(\omega)=\frac{K_{GH}}{1-WK_{GH}}, \tag{B.2}$$

where K_{GH} is given by (12). Moreover

$$W=\frac{3\mathrm{i}a\kappa(R_1-R_2)}{b^3\omega(\eta_1 Z_1-\eta_2 Z_2)}\left(\frac{M_1}{K_{G1}}-\frac{M_2}{K_{G2}}\right),$$

$$R_1=\frac{(K_{G1}-K_m)(3K_{G2}+4\mu_m)}{K_{G2}(3K_{G1}+4\mu_m)+4\mu_m(K_{G1}-K_{G2})S_1},$$

$$R_2=\frac{(K_{G2}-K_m)(3K_{G1}+4\mu_m)}{K_{G2}(3K_{G1}+4\mu_m)+4\mu_m(K_{G1}-K_{G2})S_1},$$

$$Z_1=\frac{1-\exp(-2\gamma_1 a)}{(\gamma_1 a-1)+(\gamma_1 a+1)\exp(-2\gamma_1 a)},$$

$$Z_2=\frac{(\gamma_2 b+1)+(\gamma_2 b-1)\exp[2\gamma_2(b-a)]}{(\gamma_2 b+1)(\gamma_2 a-1)-(\gamma_2 b-1)(\gamma_2 a+1)\exp[2\gamma_2(b-a)]},$$

$$\gamma_j=\sqrt{\mathrm{i}\omega\eta_j/(\kappa K_{Aj})},$$

$$K_{Aj}=\frac{K_m}{K_{Gj}}M_j,\quad j=1,2, \tag{B.3}$$

where K_s is the bulk modulus of the grains, K_{fj} are the bulk moduli of the fluids, η_j are the fluid viscosities, K_{Gj} are the Gassmann moduli given by (7) and the parameter M_j is given by (3). Finally, the approximate transition frequency separating the relaxed and unrelaxed states is given by

$$\omega_c=\frac{2\kappa K_{E2}}{\eta_2(b-a)^2} \tag{B.4}$$

(Dutta and Seriff, 1979; Carcione et al., 2003), where K_{E2} is the effective P-wave bulk modulus of the water saturated region (the subindex 2 refers to water) given by (4). We should be aware of the limitations of the theory. For simplicity in the calculations, White considered an outer sphere of radius b ($b>a$), instead of a cube. Thus, the system consists of two concentric spheres, where the volume of the outer sphere is the same as the volume of the original cube. The outer radius is $b=l/(4\pi/3)^{1/3}$, where l is the size of the cube. The distance between pockets is l. When $a=l/2$ the gas pockets touch each other. This happens when $S_1=\pi/6=0.52$. Therefore, for values of the gas saturation higher than this critical value, or values of the water saturation between 0 and 0.48, the theory is not rigorously valid. Another limitation to consider is that the size of gas pockets should be much smaller than the wavelength, i.e., $a \ll c_r/f$, where c_r is a reference velocity and f is the frequency.

Appendix C. Johnson's model of patchy saturation

In recent studies (Johnson, 2001; Müller and Gurevich, 2005) a generalization of White's model for patches of arbitrary shape was developed. In particular, Johnson developed a simple model for the dynamic bulk modulus $\tilde{K}(\omega)$, which describes the crossover between the two frequency limits. In addition to the usual parameters of Biot theory, this model has two other parameters, depending on the patch geometry. These parameters are S/V, the ratio of the surface area of a patch to the sample volume, and T, which is related to the geometry of the patches. These two parameters appear in the expressions for the high and low frequency limits. Because it is assumed that the saturation of the pore space occurs in patches, there are two different fluids that occupy different regions, which shall be denoted as regions 1 and 2. Because $\tilde{K}(\omega)$ is the Fourier transform of a real-valued response

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function, it is constrained by the reflection symmetry

$$\tilde{K}(-\omega^*) = \tilde{K}^*(\omega). \tag{C.1}$$

Moreover, general considerations on the expression for the dynamic bulk modulus allows us to infer that $\tilde{K}(\omega)$ must have the following expansions:

$$\lim_{\omega\to 0}\tilde{K}(\omega) = K_{GW}[1 - \mathrm{i}\omega T + \cdots],$$

$$\lim_{\omega\to\infty}\tilde{K}(\omega) = K_{GH}[1 - G(-\mathrm{i}\omega)^{-1/2} + \cdots] \tag{C.2}$$

(Johnson, 2001). The real valued coefficient G is

$$G = \left|\frac{\Delta p_f}{P_e}\right|^2 \frac{S}{V}\sqrt{D^*}, \tag{C.3}$$

where the effective diffusivity D^* is

$$D^* = \left[\frac{\kappa K_{GH}}{\eta_1\sqrt{D_1} + \eta_2\sqrt{D_2}}\right]^2. \tag{C.4}$$

The resulting discontinuity in pore pressure Δp_f, relative to the applied external stress P_e, is constant along the interface between the two regions and is given by

$$\frac{\Delta p_f}{P_e} = \frac{(R_2+Q_2)E_{G1} - (R_1+Q_1)E_{G2}}{\phi S_1 K_{G1} E_{G2} + \phi S_2 K_{G2} E_{G1}}. \tag{C.5}$$

S_j is the saturation of the j-th fluid, the diffusivity D_j is given by (1), the coefficients R_j, Q_j are given by (2) and the Gassmann moduli of the j-th region K_{Gj} and E_{Gj} are given, respectively, by (7) and (6). Pride et al. (1993) suggested for $\tilde{K}(\omega)$ the following expression:

$$\tilde{K}(\omega) = K_{GH} - \frac{K_{GH} - K_{GW}}{1 - \xi + \xi\sqrt{1 - \mathrm{i}\omega\tau/\xi^2}}, \tag{C.6}$$

where consistency with Eqs. (C.1) and (C.2) requires

$$\tau = \left[\frac{K_{GH} - K_{GW}}{K_{GH} G}\right]^2 \tag{C.7}$$

and

$$\xi = \frac{K_{GH} - K_{GW}}{2K_{GW}}\left(\frac{\tau}{T}\right). \tag{C.8}$$

Because the parameters ξ and τ are calculated from S/V and T separately, they are not fitting parameters; rather, they have a precise physical significance: ξ is a shape parameter, whereas τ sets the frequency scale. When $\xi < 1$ the crossover region is quite broad, whereas when $\xi > 1$ it is quite narrow. Finally, the concepts of high- and low-frequency limits allow a more reasonable definition of the transition frequency separating the relaxed and unrelaxed states. It may be defined as the frequency where the high and low frequency asymptotes for the attenuation $|\mathrm{Im}(\tilde{K}(\omega))/\mathrm{Re}(\tilde{K}(\omega))|$ intersect:

$$\omega_c = \left(\frac{G}{2T}\right)^{1/3}. \tag{C.9}$$

It is clear that the parameter S/V depends on the shape of the patches. However, the parameter T depends on the geometry of the patches in a complicated and non-local way, which can be solved only with certain simplifying geometries. Essentially, T is governed by the mean size of the patch (Johnson, 2001). Let us consider, as in the previous section, the concentric spherical geometry (White, 1975), wherein region 1 is a gas-filled sphere of porous medium of radius a surrounded by region 2 of outer water-filled sphere of radius b ($a < b$). The two Johnson's parameters have the following expression:

$$S/V = 3\frac{a^2}{b^3},$$

$$T = \frac{K_{GW}\phi^2}{30\kappa R_b^3}\{[3\eta_2 g_2^2 + 5(\eta_1 - \eta_2)g_1 g_2 - 3\eta_1 g_1^2]a^5 - 15\eta_2 g_2(g_2 - g_1)a^3 b^2 + 5g_2[3\eta_2 g_2 - (2\eta_1 - \eta_2)g_1]a^2 b^3 - 3\eta_2 g_2^2 b^5\},$$

where

$$g_j = \frac{(1 - K_m/K_s)(1/K_W - 1/K_{fj})}{1 - K_m/K_s - \phi K_m/K_s + \phi K_m/K_W}, \quad j = 1, 2. \tag{C.10}$$

Analogously, if region 1 is a layer of thickness $d_1 = 2l_1$ and region 2 is a layer of thickness $d_2 = 2l_2$ periodically separated as in White et al. (1975), we have

$$S/V = \frac{1}{l_1 + l_2},$$

$$T = \frac{K_{GW}\phi^2}{6\kappa(l_1 + l_2)}\{\eta_1 g_1^2 l_1^3 + 3\eta_1 g_1 g_2 l_1^2 l_2 + 3\eta_2 g_1 g_2 l_1 l_2^2 + \eta_2 g_2^2 l_2^3\} \tag{C.11}$$

(Johnson, 2001). The physical significance of the parameter T can be more easily understood when one phase "gas" is much more compressible and much less viscous than the other phase "fluid", i.e., as if the gas phase may be approximated as a vacuum. In this special case T represents the diffusion time for equilibrating stress in the porous skeleton over the size of the fluid patch. In other words, it is the mean lifetime for diffusion across the fluid patch (Johnson, 2001).

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