



On optimal NMO and generalised Dix equations for velocity determination and depth conversion



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ABSTRACT

The classical Earth model used to process seismic data assumes plane layers and a velocity analysis based on a hyperbolic approximation of the reflection events, where basically one parameter (the stacking velocity) is used to perform the normal move-out (NMO) correction to obtain the stacked section. We explore the possibility of using alternative approaches, other than the standard Dix equation based on the root-mean-square (RMS) velocities, to obtain the interval velocities and perform the depth conversion in order to locate the interfaces. Specifically, we consider traveltimes equations as a function of offset using different NMO approximations, based on the average, RMS and root-mean-quartic (RMQ) velocities. A generalised form of Dix's equation is used for this purpose. Moreover, we analyse the model-dependency of the different NMO equations.

We consider a simple 4-layer model and a model based on data from the Cooper basin, South Australia, to test the NMO equations. We build an elastic-velocity model and compute a common midpoint (CMP) synthetic seismogram. The reflection events are identified and traveltimes are picked to perform a non-linear inversion with the conventional (one parameter) hyperbolic approximation, the 3-term Taner and Koehler equation and approximations based on the average, RMQ and RMS velocities, also using two parameters. The model has a velocity inversion which poses a challenge to the approximations. The performance of the approximations is model dependent, so a-priori information of the velocity profile can be useful to perform a suitable inversion, or an optimal stack of the reflection events is required to test the NMO correction. Moreover, the results show that the inversion based on the RMS velocities yields better results than those based on the RMQ velocities, but this is not always the case. On the other hand, the inversion using the average velocity performs a worse velocity–depth estimation, even compared to the RMS results from the hyperbolic approximation.

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1. Introduction

The hyperbolic relation between two-way traveltimes and offset is a rough approximation to compute the NMO correction in the case of multiple plane layers and large offsets. This therefore requires a non-hyperbolic approximation. [Blais \(2007\)](#) provides various expressions and analyses their performance for the calculation of the interval velocities. He also introduces new NMO approximations. Two of them include average velocity as one of the parameters.

Although the subject has been making significant progress during the last two decades, mainly considering the anisotropic case ([Alkhalifah, 2000, 2011](#); [Alkhalifah and Tsvankin, 1995](#); [Blais, 2009](#); [Tsvankin and Grechka, 2011](#)), still some issues are not clear in

simple cases, such as plane layered isotropic media, and precise, unambiguous answers cannot be found in the literature. As [Blais \(2007\)](#) indicates, there are two main issues related to the NMO velocity analysis, i.e., the accuracy of the NMO approximation and the interval-velocity estimation and related depth conversion. Optimal traveltimes approximations can be found for instance in [Causse \(2004\)](#), [Fomel and Stovas \(2010\)](#). In particular, these authors propose a five-parameter non-hyperbolic moveout approximation that reduces to known equations, some of them given here, with a particular choice of the parameters. However, a good NMO approximation should also be useful to perform an accurate velocity inversion and depth conversion. [Blais \(2007\)](#) provides such approximations, stating: “Two of the new NMO approximations include average velocity as one of the parameters. This enables an estimate of reflector depth directly from velocity analysis rather than depth estimation through the Dix formula and RMS velocities”. Here, we use one of [Blais'](#) approximation and pose the question: Is average velocity better to estimate the interval velocities and interface depth than

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Table 1
4-Layer acoustic model.

Layer	Base of layer (km)	P velocity (km/s)
1	0.4	2
2	0.7	3
3	0.8	3.5
4	1.3	4

RMS velocities? Moreover, according to [Blais \(2007\)](#): “The new approximations appear to be the most accurate in terms of residual traveltimes and RMS velocity estimations, particularly at large offsets”. A second question to answer is: Are there NMO approximations that can perform better in general or their performances depend on the depth–velocity model?

In order to answer these questions and test the NMO approximations, we apply them to a CMP seismogram generated from Tirrawarra well data in the Cooper basin ([Laws and Gravestock, 1998](#)). Preliminary work based on this model and focused on the 3-term equation has been performed by [Qadrouh \(2008\)](#). [Blais \(2007\)](#) considered several NMO approximations constrained by the condition that the traveltime and its two derivatives with respect to the offset squared at zero offset be the same as those of the exact traveltime function. Here, we show that these approximations are model-dependent by also computing residual traveltimes for a simple model 4-layer model. Then, we obtain the interval velocities for the Tirrawarra model from the RMS and average velocities. In the inversion procedure, the two-way zero-offset traveltime is assumed to be known, resulting in one parameter, using the hyperbolic approximation (the stacking or NMO velocity) and two parameters, using the 3-term Taner and Koehler equation and one of [Blais’s](#) equations based on the average velocity.

2. Reflection-traveltime approximations

Let us consider a multilayered Earth consisting of n layers. [Bolshykh \(1956\)](#) and [Taner and Koehler \(1969\)](#) derived the first power series for the reflection traveltime from the base of the n th layer, located at depth z . The result is

$$T^2 = \sum_{i=0}^{\infty} c_i x^{2i}, \tag{1}$$

where c_i are coefficients depending on the model parameters and x is the offset.

We consider at most three terms:

$$T = \sqrt{T_0^2 + \frac{x^2}{a_2} + \frac{a_2^2 - a_4}{4T_0^2 a_2^4} x^4}, \tag{2}$$

Table 2
Tirrawarra (Cooper basin) model.

Layer	Base of layer (km)	P velocity (km/s)	h (km)	T_0 (s)	V_1 (km/s)	V_2 (km/s)	V_4 (km/s)
1	0.413	1.59	0.413	0.5195	1.59	1.59	1.59
2	0.714	2.023	0.301	0.8171	1.748	1.760	1.785
3	1.504	2.	0.790	1.6071	1.872	1.882	1.90
4	2.098	2.129	0.594	2.1651	1.938	1.949	1.967
5	2.428	3.132	0.330	2.3758	2.044	2.081	2.170
6	2.676	3.223	0.248	2.5297	2.116	2.168	2.287
7	3.	5.250	0.324	2.6531	2.261	2.4	2.80
8	3.334	3.587	0.334	2.8394	2.348	2.496	2.874
9	3.793	4.787	0.459	3.0311	2.503	2.699	3.140

where T_0 is the two-way zero-offset traveltime to the base of the n layer, x is the offset,

$$a_j = \frac{1}{T_0} \sum_{k=1}^n v_k^j t_k, \quad V_{jn} = a_j^{1/j}, \tag{3}$$

are the velocity moments, v_k is the seismic velocity of the k layer, t_k is the vertical two-way traveltime within layer k , V_{2n} is the RMS velocity, V_4 is the RMQ velocity and $T_0 = \sum^n t_i$. Interface k is intended to be the base of layer k in the following. The average velocity is

$$V_{1n} = \frac{2z}{T_0} = a_1 = \frac{1}{T_0} \sum_{k=1}^n v_k t_k. \tag{4}$$

Other NMO approximations reported by [Blais \(2007\)](#), are obtained by keeping time T and its two derivatives with respect to x^2 at $x = 0$ the same as those of the exact traveltime function $T(x)$. These approximations are given in Eqs. (5)–(9) below.

Let us consider the n th interface. [Malovichko \(1978\)](#) derived the shifted hyperbola approximation,

$$T = T_0 \left(1 - \frac{1}{s}\right) + \frac{1}{s} \sqrt{T_0^2 + \frac{sx^2}{V_{2n}^2}}, \quad s = \frac{a_4}{a_2^2} = \left(\frac{V_{4n}}{V_{2n}}\right)^4. \tag{5}$$

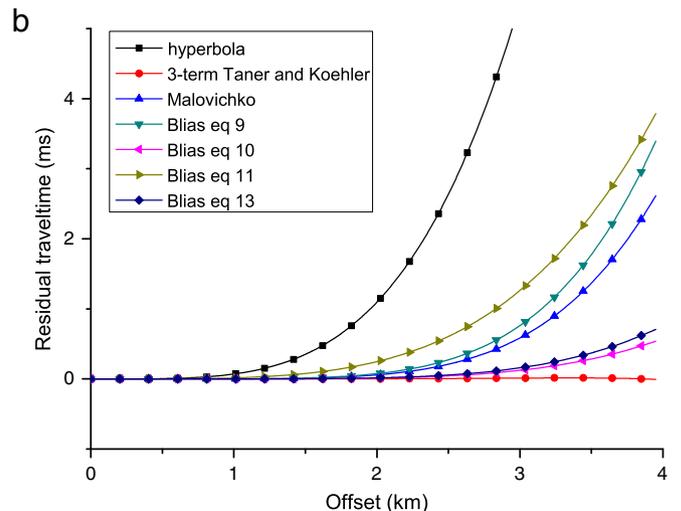
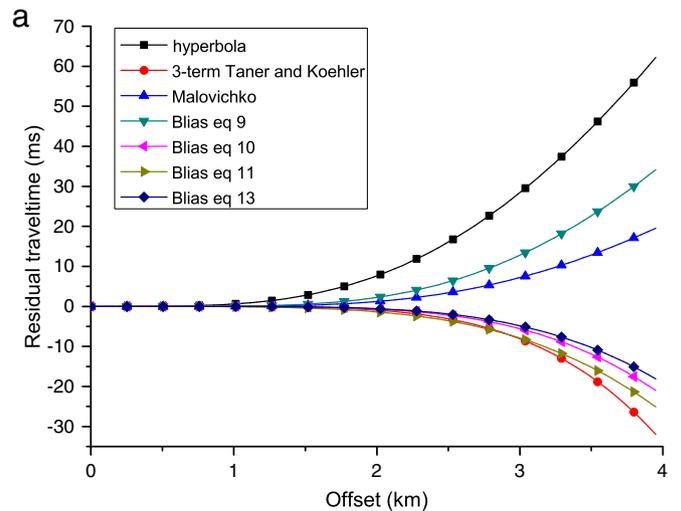


Fig. 1. Traveltime residuals for different NMO approximations corresponding to the deepest interfaces of the 4-layer (a) and Cooper basin (b) models.

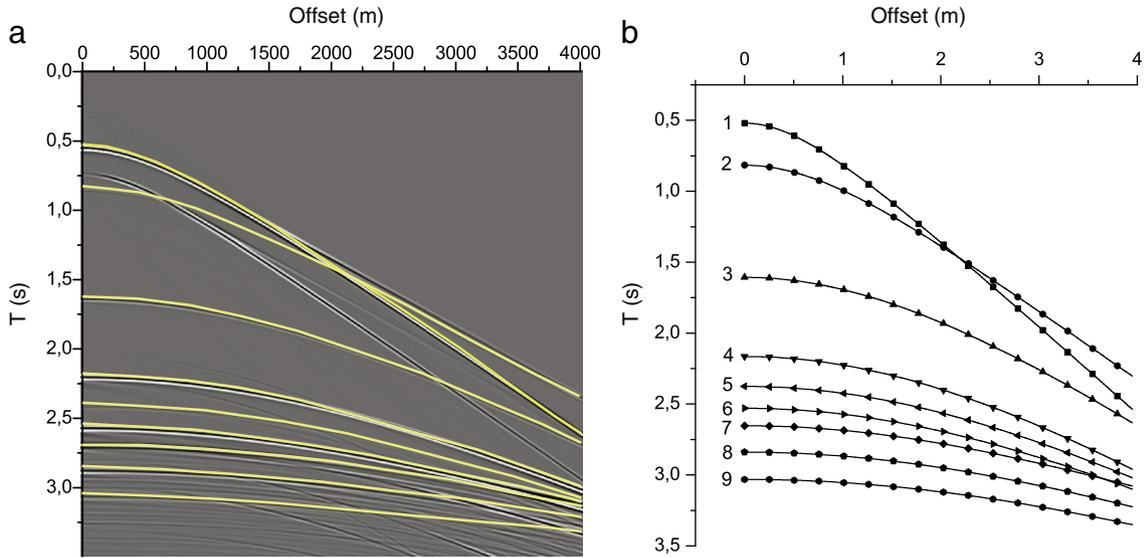


Fig. 2. Synthetic CMP data (a) and traveltime picks (b) corresponding to the Cooper basin model. Primary reflection events are highlighted. The numbers indicate the interfaces listed in Table 2.

A three-term NMO approximation, similar to the one derived by Alkhalifah and Tsvankin (1995) is

$$T = \sqrt{T_0^2 + \frac{x^2}{V_{2n}^2} - \frac{(s-1)x^4}{V_{2n}^2[4T_0^2V_{2n}^2 + (3+s)x^2]}} \quad (\text{Bias Eq. (9)}). \quad (6)$$

An approximation derived from Taner et al. (2005) is

$$T = \sqrt{T_0^2 + \frac{x^2}{(V_{2n} + ax)^2}}, \quad a = \frac{s-1}{8T_0^2V_{2n}} \quad (\text{Bias Eq. (10)}). \quad (7)$$

A relation based on the average velocity is

$$T = \sqrt{\left(T_0^2 + \frac{x^2}{V_{1n}^2}\right) \left(1 + \frac{gx^2}{T_0^2V_{1n}^2(1+g)}\right)^{-1}}, \quad g = \frac{V_{2n}^2}{V_{1n}^2} - 1 \quad (\text{Bias Eq. (11)}). \quad (8)$$

Finally, Bias (2007) considers another approximation,

$$T = \sqrt{T_0^2 + \frac{x^2}{V_{2n}^2 + bx^2}}, \quad b = \frac{s-1}{4T_0^2} \quad (\text{Bias Eq. (13)}). \quad (9)$$

3. Exact reflection traveltime

To test these approximations, we compute the traveltime residuals $T - T_e$, where T_e is the exact traveltime. This is computed as

$$x = p \sum_k \frac{v_k^2 t_k}{\sqrt{1-p^2v_k^2}}, \quad T_e = \sum_k \frac{t_k}{\sqrt{1-p^2v_k^2}}, \quad (10)$$

(Bolshykh, 1956; Pilant, 1979), where $p = \sin\theta / v_n$ is the ray parameter and θ is the incidence angle on the first (upper) interface.

4. Generalised Dix equation

It can be shown that to obtain the interval velocities v_k , we may use the following generalisation of the Dix equation

$$v_k = \left(\frac{V_{jk}^j T_k - V_{j(k-1)}^j T_{k-1}}{T_k - T_{k-1}} \right)^{1/j}, \quad (11)$$

where

$$T_k = \sum_{l=1}^k t_l, \quad T_n = T_0. \quad (12)$$

The case $j = 2$ is the standard Dix's equation (Dix, 1955). The other two cases yield the interval velocities from the average velocity ($j = 1$) and from v_{4n} ($j = 4$).

5. Time to depth conversion

From the interval velocities one can compute the depth of the interfaces as

$$z_k = \frac{1}{2} \sum_{l=1}^k v_l t_l, \quad (13)$$

where t_l is the two-way traveltime corresponding to layer l , given by

$$t_1 = T_{01} \quad \text{and} \quad t_l = T_{0l} - T_{0(l-1)}, \quad l = 2, \dots, n, \quad (14)$$

where T_{0l} is the two-way traveltime corresponding to interface l .

Table 3

Cases to obtain the interval velocities.

Equation	Velocity
Hyperbola	$V_{2k} = 1/\sqrt{p_1}$
3-term TK	$V_{2k} = 1/\sqrt{p_1}$
3-term TK	$V_{4k} = \left((1-4p_2T_{0k}^2/p_1^2) \right)^{1/4} / \sqrt{p_1}$
Bias Eq. (10)	$V_{2k} = p_1$
Bias Eq. (10)	$V_{4k} = p_1(1 + 8p_1p_2T_{0k}^2)^{1/4}$
Bias Eq. (11)	$V_{1k} = 1/\sqrt{p_1}$
Bias Eq. (11)	$V_{2k} = 1/\sqrt{p_1 - p_2T_{0k}^2}$

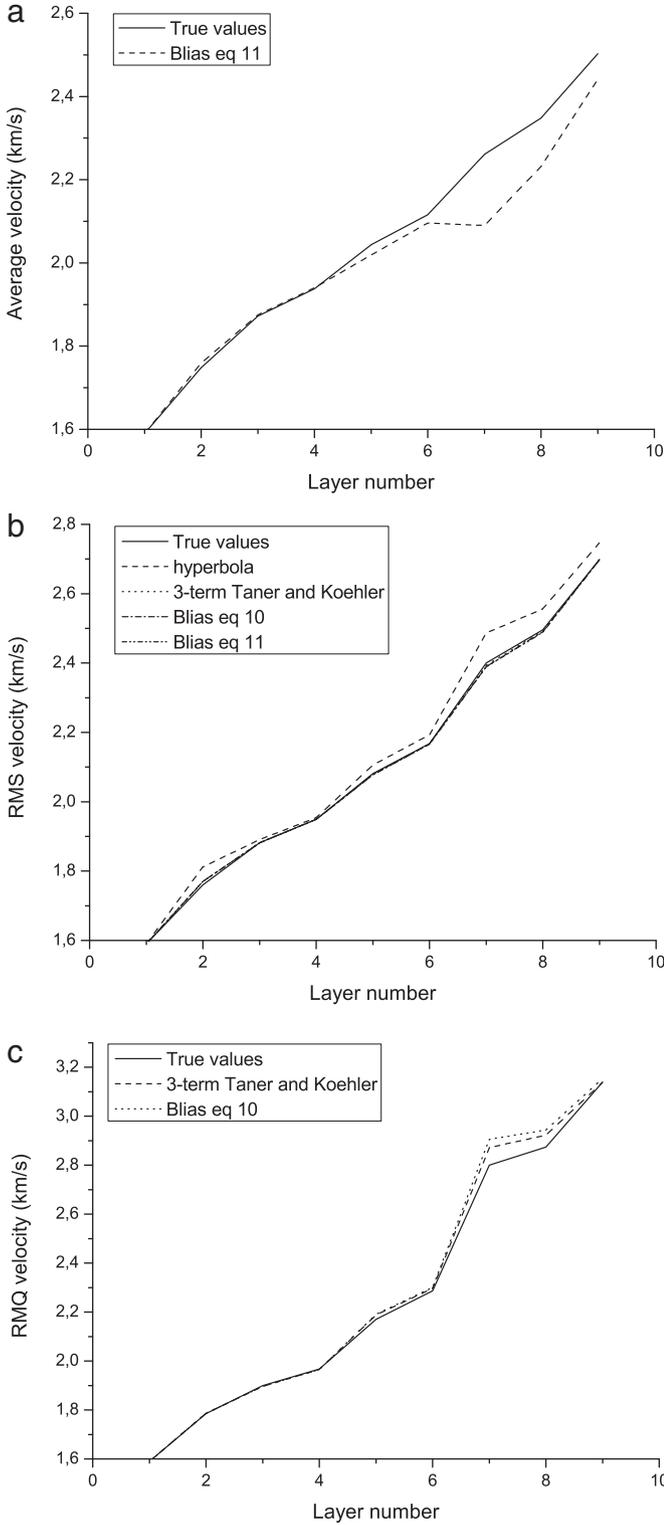


Fig. 3. Average (a), RMS (b) and RMQ (c) velocities obtained from the inversion compared to the true values.

6. Examples

The different NMO equations are verified by using the models shown in Tables 1 and 2. Table 2 provides also the average velocity (V_1), the RMS velocity (V_2) and the RMQ velocity (V_4). The second model corresponds to the Tirrawarra well in the Cooper basin, South Australia. Fig. 1a and b show the hyperbolic (black line) and non-hyperbolic residual traveltimes as a function of offset, corresponding to the deepest

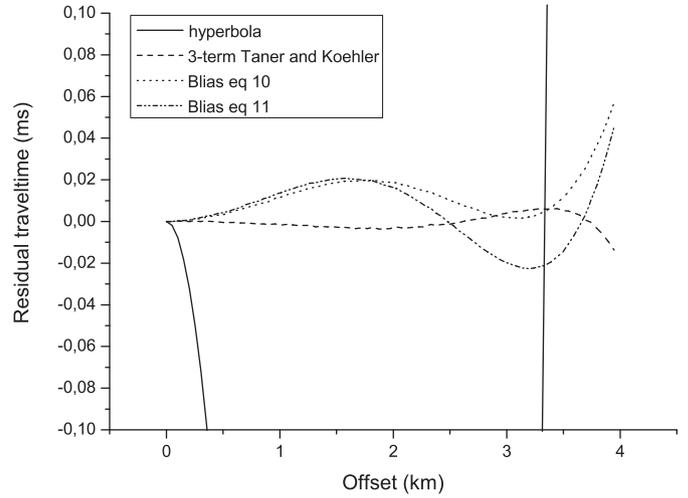


Fig. 4. Traveltime residuals corresponding to the deepest interface of the Cooper basin model.

interfaces of the models given in Tables 1 and 2, respectively. The curves clearly show that the approximations are model dependent. In the first case, Bias Eqs. (10) and (13) show the best performance, while the Taner and Koehler 3-term equation and Bias Eq. (10) are the best for the model displayed in Table 2. The hyperbolic approximation is the worst in both cases. It can be shown that the 3-term Taner and Koehler equation provides the best fit for interfaces 4–9 of the second model. The fact that this NMO equation is the best for model 2 does not mean that it can perform equally well in other cases. Bias (2009) has also shown that the NMO approximations are model-dependent. Therefore, a-priori knowledge of the velocity trend is important to establish the best NMO approximation to obtain the interval velocities. Alternatively, the equation providing the best fit should be used to obtain the velocities.

A synthetic CMP gather has been computed with a full-wave solver based on the pseudospectral method (Carcione, 2007; Seriani et al., 1992) (see Fig. 2a). We have assumed that the S-wave velocities are given by $v_s = v_p / \sqrt{3}$ (a Poisson medium) and the density is computed from Gardner's relation $\rho = 1741 v_p^{0.25}$ (Mavko et al., 1998, p. 254), where v_p is given in km/s and ρ in kg/m^3 . The P-wave velocity below interface 9 is set to 5.2 km/s. The source – a Ricker wavelet – has a dominant frequency of 25 Hz. We have used 561×561 grid points with a uniform grid spacing of 10 m along the horizontal and vertical directions and the time step is 1 ms.

Next, we compute the Cooper-basin interval velocities from CMP data, using the hyperbola, Taner and Koehler 3-term formula and Bias Eqs. (10) and (11). Let us assume for simplicity that we have obtained the exact traveltimes picks from the CMP gather. These are shown in Fig. 2b. Actually, a PS event from the first interface can be seen between events 1 and 2, showing that this visual pick to identify PP reflections is not always reliable. We have L pairs (x_i, T_i) (offset-traveltime) to fit with the hyperbolic approximation and Eqs. (2) and (8), respectively. It is a non-linear minimisation problem of the functions

$$\begin{aligned} & \sum_{i=1}^L \left(\sqrt{T_0^2 + p_1 x_i^2} - T_i \right)^2 \text{ hyperbola} \\ & \sum_{i=1}^L \left(\sqrt{T_0^2 + p_1 x_i^2 + p_2 x_i^4} - T_i \right)^2 \text{ 3-term Taner and Koehler} \\ & \sum_{i=1}^L \left(\sqrt{T_0^2 + \frac{x_i^2}{(p_1 + p_2 x_i^2)^2}} - T_i \right)^2 \text{ Bias Eq. (10),} \\ & \sum_{i=1}^L \left(\sqrt{\frac{T_0^2 + p_1 x_i^2}{1 + p_2 x_i^2}} - T_i \right)^2 \text{ Bias Eq. (11),} \end{aligned} \tag{15}$$

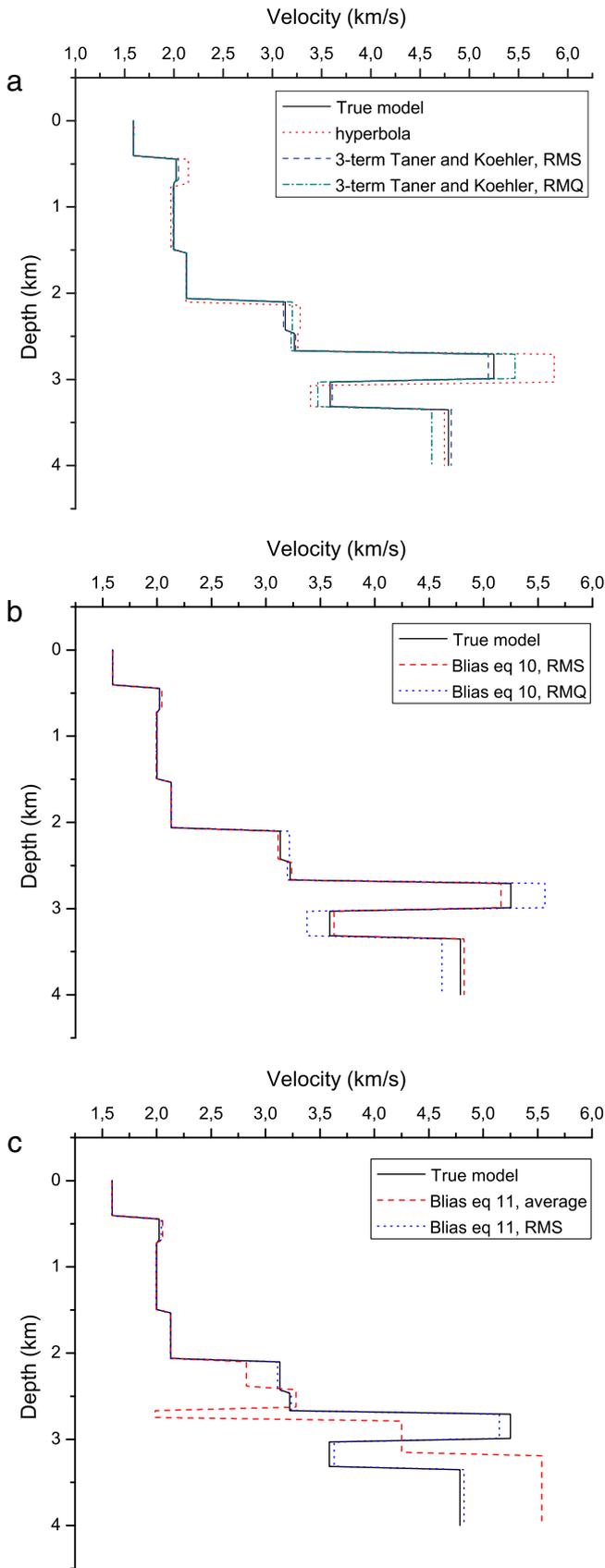


Fig. 5. Inversion results based on the hyperbolic equation (conventional velocity analysis) and the 3-term Taner and Koehler equation (a), Bias Eq. (10) (b) and Bias Eq. (11) (c).

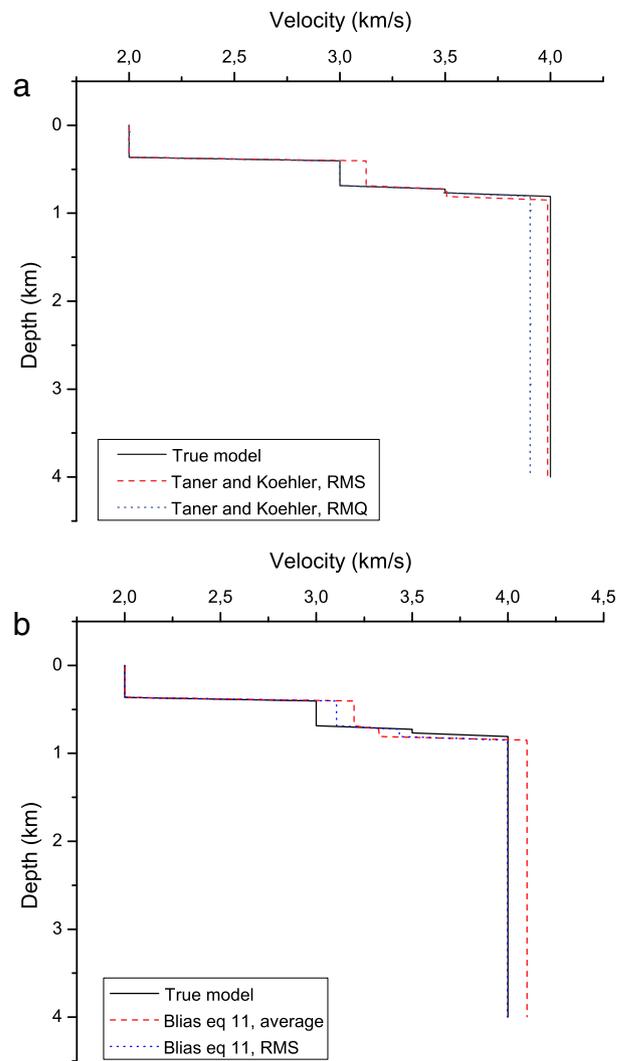


Fig. 6. Inversion results for the 4-layer model, based on the 3-term Taner and Koehler equation (a) and Bias Eq. (11) (b).

respectively, to obtain the parameters p_1 and p_2 (T_0 is known). The minimisation algorithm, called Praxis, is illustrated in Chapter 7 of Brent (1971).

We consider several cases, listed in Table 3, to compute the interval velocities, where TK denotes Taner and Koehler. The estimation of the average (a), RMS (b) and RMQ (c) velocities, compared to the true values, are shown in Fig. 3. The average velocity deviates from the true values at the deepest layers starting from the highest velocity. On the other hand, the 3-term Taner and Koehler approximation performs better than the hyperbola, as expected. Also, the two Bias RMS velocities obtained from Eqs. (7) and (8) yield a good agreement. Similarly, the RMQ velocity from Bias Eq. (10) is a good approximation to the true one as can be seen in Fig. 1b. Fig. 4 shows the traveltimes residuals computed with the inversion coefficients p_1 and p_2 . The curve out of bounds corresponds to the conventional NMO approximation.

The inversion results (interval velocity and interface depth) are shown in Fig. 5. The conventional velocity analysis underestimates the highest velocity by approximately 0.5 km/s. The 3-term Taner–Koehler equation gives the best estimation in this case, since this approximation is the best as can be seen in Fig. 1b. The RMS velocities are the most reliable ones while the average velocity fails to estimate the highest velocity, where there is a velocity inversion and does not perform better than the hyperbola, despite the fact that the traveltimes equation (Bias Eq. (11)) provides a better approximation to the true event, as shown

in Fig. 1, and therefore a better stack. In all the cases, the RMS velocities seem to handle better the velocity inversion.

To show that the inversion is model dependent, we display in Fig. 6 the results for the 4-layer model, using the 3-term Taner and Koehler equation and Bias Eq. (11). The RMS velocities from both equations perform very similar, with a slightly better performance of the latter, since its residual move out is closer to zero than that of the first approximation (see Fig. 1). On the other hand, the RMQ velocities in Fig. 6a (dots) performs better than the RMS velocities, with the only exception of the last layer. The inversion obtained from the average velocity is better than that of the more complex model but it is still worse than that obtained with the RMS velocity.

7. Conclusions

Optimal NMO approximations are required to obtain accurate images of the subsurface (optimal stacking), compute reliable values of the interval velocities and perform the conversion from two-way traveltimes to depth. It is shown here that a-priori knowledge of the velocity profile in the area is useful, since the NMO equations are model-dependent. Alternatively, tests are necessary to identify the optimal NMO equation providing maximum stacking of the reflection events. We have considered a particular velocity model from the Copper basin, obtained the traveltimes picks of the reflection events and performed an inversion of the root-mean and average velocities using different NMO approximations. It has been found that the root-mean-squared velocities provide a better estimation of the interval velocities than the average and root-mean-quartic velocities, and therefore a better depth conversion, but this is not general, since the opposite performance may occur depending on the model. Moreover, it is clear that higher order (non-hyperbolic) approximations perform much better than the classical hyperbolic equation used in conventional processing. The analysis presented here can be extended to dipping interfaces and to the case of anisotropic layers, where even in the case of a single homogeneous layer the traveltimes are not described by a hyperbolic function and therefore non hyperbolic approximations are more important.

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Appendix A. Generalised Dix equation

Following Stovas (2008), we demonstrate Eq. (11) using the continuum. First, note that a_j in Eq. (3) can be expressed as

$$a_j = \frac{1}{T_0} \int_0^{T_0} v^j dt = \frac{2}{T_0} \int_0^H v^{j-1} dz, \quad (16)$$

where H is the thickness of the layer (or set of layers in the discrete case). We note that

$$a_j = \frac{2}{T_0} I_{j-1}, \quad (17)$$

with I_j defined in Eq. (A-1) by Stovas (2008) and that $a_0 = 1$.

Taking the derivative of $T_0 a_j$ as follows and using Eq. (16), we obtain the generalised Dix equation

$$\frac{d(T_0 a_j)}{dT_0} = v^j, \quad (18)$$

which is the continuum version of Eq. (11). Eq. (18) is equivalent to Eq. (B-7) in Stovas (2008), since S_k in his Eq. (A-2) is given by $S_k = a_{2k}/a_2^k$, such that $S_2 = s$ (see Eq. (5)).

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