Effect of capillarity and relative permeability on $Q$ anisotropy of hydrocarbon source rocks

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SUMMARY
Shale reservoir formations are porous rocks of low permeability composed of fluid-saturated illite–smectite and kerogen layers, which behave as viscoelastic transversely isotropic (VTI) media at long wavelengths, that is, much larger than the average layer thickness. Seismic waves travelling across these heterogeneous materials induce wave-induced fluid flow (WIFF) and Biot slow waves generating energy loss (mesoscopic loss) and velocity dispersion. When these formations are saturated by two-phase fluids, the presence of capillary forces—interfacial tension—and interaction between the two fluids as they move within the pore space need be taken into account. This can be achieved using a Biot model of a poroelastic solid saturated by a two-phase fluid that includes capillary pressure and relative permeability functions and supports the existence of two slow waves. An upscaling finite-element method is used to analyse the WIFF, which determines an effective VTI medium predicting higher attenuation and $(Q)$ anisotropy than the classical single-phase (single fluid) models.

Key words: Numerical approximations and analysis; Numerical modelling; Numerical solutions; Computational seismology; Seismic anisotropy; Seismic attenuation.

1 INTRODUCTION
The purpose of this work is to analyse the anisotropy in seismic attenuation of shale reservoir rocks as a function of fluid saturation and spatial distribution of organic matter (oil and kerogen) in the rock matrix. Most shale reservoir rocks are laminated media (with typical thickness of mm) of very low permeability composed of illite–smectite layers and organic matter in the form of oil, gas and kerogen. For seismic wavelengths much larger than the thickness of the layers, these laminated materials behave as homogeneous viscoelastic transversely isotropic (VTI) media.

Biot (1956a,b; 1962) developed a theory to describe wave propagation in a poroelastic solid saturated by a single-phase fluid (a single-phase Biot medium—SPBM). The theory predicts the existence of two compressional waves (one of them slow), and one shear wave. The fast $P$ wave has solid and fluid motions in phase, and the slow Biot $P$ wave has out-of-phase motion, causing strong energy losses. The existence of the second slow wave was confirmed by Plona (1980). Most recently, Bouzidi & Schmitt (2009) presented experiments where the slow P2 wave was observed at a wide range of incident angles.

However, Biot’s theory does not take into account the presence of capillary forces and interference effects between the two fluids as they move within the pore space. A generalization of Biot’s theory to the case when a poroelastic medium is saturated by a two-phase fluid (a two-phase Biot medium—2PBM) was presented in Santos et al. (1990a,b) and Ravazzoli et al. (2003). The 2PBM model includes effects of capillary and relative permeability functions defined in terms of the two-phase Darcy’s law (Scheidegger 1974; Peaceman 1977). The model predicts the existence of one fast wave, two slow compressional waves and one shear wave. Capillary forces are responsible for the existence of one additional slow wave, while relative permeability functions induce energy losses due to interferences between the two-phase fluids as they move within the pores. The work by Müller & Sahay (2011) presents an extension of Biot’s theory for single-phase fluids predicting the existence of a second slow shear wave.

Among others, Dutta & Odé (1979), Mochizuki (1982), Berrymann et al. (1988) and Toksöz et al. (1976) tackled the analysis of the quasi-static and dynamic behaviour of porous rocks with partial, miscible or segregated fluid saturation. None of the above approaches incorporate the capillary forces. Using a homogenization approach, Auriault (1989) included a description of capillary effects at the pore scale obtaining a two-phase Darcy law.

One significant cause of attenuation in layered fluid-saturated poroelastic media is wave-induced fluid flow (WIFF), by which the
fast compressional ($P$) and shear ($S$) waves are converted to slow (diffusive) Biot waves as they travel across regions with heterogeneities in the fluid and petrophysical properties of the medium. We refer to this mechanism as mesoscopic loss envisioning the length scale of the heterogeneities to be larger than the grain sizes but much smaller than the wavelength of the pulse. For instance, if the matrix porosity varies significantly from point to point, diffusion of pore fluid between different regions constitutes a mechanism that can be important at seismic frequencies. Pride et al. (2004) have demonstrated the importance of the mesoscopic effects in the context of exploration geophysics as being the dominant $P$-wave attenuation mechanism in reservoir rocks at seismic frequencies.

A review of the different theories and authors, who have contributed to the understanding of this mechanism, can be found, for instance, in Carcione & Picotti (2006), Müller et al. (2010) and Carcione (2014). In this work, the analysis of the WIFF takes into account the presence of two slow waves and the additional energy losses present in the case of two-phase fluids.

For an analysis of anisotropy in stratified media, we mention the early work by Carcione et al. (1991), whereas Carcione et al. (2011) treated the specific case of source rocks without energy loss. Gelin et al. (2004) and Carcione (2014). In this work, the analysis of the WIFF takes into account the presence of two slow waves and the additional energy losses present in the case of two-phase fluids.

Qi et al. (2014) studied the effects of capillarity on attenuation and dispersion in isotropic patchy-saturated rocks and found that the capillary action leads to an additional stiffening and thereby to higher phase velocities, with weakening diffusion process and attenuation.

The work in Santos & Carcione (2015) uses the SPBM model to define a set of five harmonic compressibility and shear experiments for determining the stiffness coefficients and the corresponding energy velocities and dissipation factors of a long-wave equivalent VTI medium to a densely fractured fluid-saturated poroelastic medium. The numerical experiments are formulated as boundary value problems (BVP) in the space–frequency domain that are solved using the finite-element (FE) method. See also Santos & Gauzellino (2017) for a detailed description of the use of the FE method in the context of numerical rock physics and upscaling.

2 THE MODEL DESCRIBING A POROELASTIC MEDIUM SATURATED BY A TWO-PHASE FLUID

In a porous solid saturated by a two-phase fluid exist wetting and non-wetting phases denoted with the subscripts (or superscripts) ‘w’ and ‘n’, respectively, while ‘s’ will indicate the solid phase. Let $S_1$ and $S_0$ be the saturation and residual saturation of the 1-phase, $l = n$, $w$, so that $S_0 < S_1 < 1$. Besides, we assume full saturation of the pore space, $S_0 + S_1 = 1$ (Scheidegger 1974; Peaceman 1977). In the shale reservoir model studied in this work, gas is always the non-wetting phase (see Fig. 1).

The relative particle fluid displacements are

$$\mathbf{u}^i = \phi(\mathbf{u}^w - \mathbf{u}^n), \quad \xi^i = -\nabla \cdot \mathbf{u}^i, \quad l = n, w,$$

where $\mathbf{u}^w = (u^w, t) \ell = n, w, \quad i = 1, 2, 3$ are the time Fourier transforms of the displacement vectors of the solid and fluid phases and $\phi$ is the matrix effective porosity.

Define $\varepsilon_i = \varepsilon_i(\mathbf{u}^w)$ and $e^i = e^i(\mathbf{u}^w)$ as the Fourier transforms of the strain tensor of the solid and its linear invariant, respectively, and set $\mathbf{u} = (\mathbf{u}^w, \mathbf{u}^n, \mathbf{u}^s)$. Let $\tau = \tau_{ij}$ and $\varepsilon = \varepsilon_{ij}, \quad i, j = 1, 2, 3$ denote the time Fourier transforms of the stress and strain tensors, respectively. Also, let $P_i$ denote the time Fourier transform of the infinitesimal change in the pressure of the f-phase, taken with respect to the reference value $\bar{P}_1 = 1 = n, w$. This reference value is associated with the initial equilibrium state having non-wetting fluid saturation $\bar{S}_w$ and porosity $\phi$. $\overline{P}_n$ and $\overline{P}_w$ are related through the capillary relation (Scheidegger 1974; Peaceman 1977):

$$P_n = \overline{P}_n(S_n + \bar{S}_w) - \bar{P}_n + P_n - (\bar{P}_w + P_w) = P_n(S_n) + P_n - P_w \geq 0. \quad (1)$$

The stress–strain relations of a 2PBM are (Santos et al. 1990a; Ravazzoli et al. 2003)

$$\tau_{ij}(\mathbf{u}) = \bar{S}_w (\bar{S}_n + \beta + \zeta) P_n - (\beta + \zeta) P_w - B_i e^i + M_i \xi^n + M_3 \xi^w, \quad (2)$$

$$\tau_{w}(\mathbf{u}) = \bar{S}_w (\bar{S}_n + \zeta) P_n - \zeta P_w - B_2 e^i + M_3 \xi^n + M_2 \xi^w, \quad (3)$$

where

$$\beta = \frac{\overline{P}_n}{P_n(S_n)}, \quad \zeta = \frac{\overline{P}_w}{P_n(S_n)}. \quad (5)$$

The coefficient $N$ is the shear modulus of the dry rock. The determination of the other coefficients in eqs (2)–(4) is explained in Santos et al. (1990a), Ravazzoli et al. (2003) and Santos & Gauzellino (2017).

The governing equations for a 2PBM in the diffusive range of frequencies are

$$\frac{\partial \tau_{ij}}{\partial \xi^j} = 0, \quad (6)$$

$$i \omega (\bar{S}_w) \frac{\eta_n}{K_{\overline{P}_n}(\bar{S}_n)} u^n_j - i \omega d_{\overline{P}_n} u^w_j + \frac{\partial T_n}{\partial x_j} = 0, \quad (7)$$

$$i \omega (\bar{S}_n) \frac{\eta_w}{K_{\overline{P}_n}(\bar{S}_w)} u^n_j - i \omega d_{\overline{P}_n} u^n_j + \frac{\partial T_n}{\partial x_j} = 0, \quad j = 1, 2, 3. \quad (8)$$

The cross dissipative coefficient $d_{\overline{P}_n}$ is taken to be

$$d_{\overline{P}_n}(\bar{S}_n, \bar{S}_w) = \epsilon \left( \frac{\eta_n}{K_{\overline{P}_n}(\bar{S}_n)} \right) \left( \frac{\eta_w}{K_{\overline{P}_n}(\bar{S}_w)} \right). \quad (9)$$
In eqs (7) and (8), \( \eta_n, \eta_w \) are the fluid viscosities and \( k, K_n(S_n), K_m(S_m) \) are the absolute and relative permeabilities, respectively. In this work, the following relative permeability and capillary pressure functions are used (Ravazzoli et al. 2003):

\[
K_m(S_m) = \left( 1 - (1 - S_m)/(1 - S_m) \right)^2,
\]
\[
K_n(S_n) = \left( 1 - S_n - S_m \right)/(1 - S_m) \right)^2,
\]
\[
P_{es}(S_m) = A \left( 1/(S_m + S_w - 1)^2 - S_m^2/(1 - S_m - S_w) \right)^2.
\]

where \( A \) is the capillary pressure amplitude, chosen to be 30 kPa.

3 THE EQUIVALENT VISCOELASTIC TRANSVERSELY ISOTROPIC MEDIUM

As shown in Krzikalla & Müller (2011), a fluid-saturated poroelastic solid with a set of horizontal layers behaves as a VTI medium with vertical symmetry axis.

Denoted by \( \sigma_{ij}(\tilde{u}) \) and \( e_{ij}(\tilde{u}) \) the stress and strain tensor components of the equivalent VTI medium, where \( \tilde{u} \) denotes the solid displacement vector at the macroscale. The corresponding stress–strain relations, stated in the space–frequency domain and assuming a closed system are (Carcione, 2014)

\[
\sigma_{11}(\tilde{u}) = p_{11} e_{11}(\tilde{u}) + p_{12} e_{22}(\tilde{u}) + p_{13} e_{33}(\tilde{u}),
\]

\[
\sigma_{22}(\tilde{u}) = p_{12} e_{11}(\tilde{u}) + p_{11} e_{22}(\tilde{u}) + p_{13} e_{33}(\tilde{u}),
\]

\[
\sigma_{33}(\tilde{u}) = p_{13} e_{11}(\tilde{u}) + p_{11} e_{33}(\tilde{u}) + p_{33} e_{33}(\tilde{u}),
\]

\[
\sigma_{23}(\tilde{u}) = 2 p_{55} e_{23}(\tilde{u}),
\]

\[
\sigma_{31}(\tilde{u}) = 2 p_{55} e_{13}(\tilde{u}),
\]

\[
\sigma_{32}(\tilde{u}) = 2 p_{66} e_{12}(\tilde{u}).
\]

In a VTI medium \( p_{12} = p_{13} = 2p_{66} \), so that only five independent stiffness, that is, \( p_{11}, p_{33}, p_{13}, p_{55} \) and \( p_{66} \) need to be considered.

Santos & Carcione (2015) have shown that the stiffnesses \( p_{ij} \) in eqs (11)–(16) can be determined using five time-harmonic experiments. Next, we present the generalization of those experiments using the 2PBM to determine a VTI medium long-wave equivalent to a fine layered poroelastic solid saturated by a two-phase fluid.

Denoting by \( x_1 \) and \( x_3 \) the horizontal and vertical coordinates, we will solve eqs (6)–(8) in the 2-D case on a reference square \( \Omega = (0, L)^2 \) with boundary \( \Gamma \) in the \((x_1, x_3)\)-plane. Set \( \Gamma = \Gamma^L \cup \Gamma^R \cup \Gamma^\tau \), where \( \Gamma^L, \Gamma^R \) and \( \Gamma^\tau \) denote the left, right, bottom and top boundaries of \( \Omega \), respectively. Denote by \( \nu \) the unit outer normal on \( \Gamma \) and let \( \chi \) be a unit tangent on \( \Gamma \) oriented counterclockwise so that \( \{ \nu, \chi \} \) is an orthonormal system on \( \Gamma \). To determine the five independent stiffness coefficients, we solve eqs (6)–(8) in \( \Omega \) with the boundary conditions:

\[
u^u \cdot \nu = 0, \quad u^w \cdot \nu = (x_1, x_3) \in \Gamma,
\]

that is, no fluids enter or leave the sample, and additional boundary conditions for each \( \nu \).

To determine \( p_{33} \), we impose the boundary conditions:

\[
\tau(u) \nu \cdot \nu = -\Delta P, \quad (x_1, x_3) \in \Gamma^\tau,
\]

\[
\tau(u) \nu \cdot \chi = 0, \quad (x_1, x_3) \in \Gamma,
\]

\[
u^u \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma \setminus \Gamma^\tau.
\]

Using the relation

\[
\frac{\Delta V(\omega)}{V} = -\frac{\Delta P(p_{33}(\omega))}{p_{33}(\omega)}
\]

where \( V \) is the original volume of the sample, \( p_{33}(\omega) \) can be determined from eq. (21) measuring the complex volume change \( \Delta V(\omega) \approx L u_{13}^{(1)}(\omega) \), where \( u_{13}^{(1)}(\omega) \) is the average of the vertical component of the solid phase at the boundary \( \Gamma^\tau \).

To determine \( p_{11} \), the following boundary conditions are used:

\[
\tau(u) \nu \cdot \nu = -\Delta P, \quad (x_1, x_3) \in \Gamma^R \cup \Gamma^\tau,
\]

\[
\tau(u) \nu \cdot \chi = 0, \quad (x_1, x_3) \in \Gamma,
\]

\[
u^u \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma \setminus \Gamma^R.
\]

Thus, this experiment determines \( p_{11} \) as indicated for \( p_{33} \), measuring the oscillatory volume change.

To determine \( p_{55} \), we apply the boundary conditions:

\[
\tau(u) \nu \cdot \nu = -\Delta P, \quad (x_1, x_3) \in \Gamma^L \cup \Gamma^\tau,
\]

\[
\tau(u) \nu \cdot \chi = 0, \quad (x_1, x_3) \in \Gamma,
\]

\[
u^u \cdot \nu = 0, \quad (x_1, x_3) \in \Gamma \setminus \Gamma^L.
\]

From eqs (11) and (13), we get

\[
\sigma_{11} = p_{11} e_{11} + p_{13} e_{33}, \quad \sigma_{33} = p_{13} e_{11} + p_{33} e_{33},
\]

with \( e_{11} \) and \( e_{33} \) being the (macroscale) strain components at \( \Gamma^L \) and \( \Gamma^R \), respectively. Since \( \sigma_{11} = \sigma_{33} = -\Delta P \) (cf. eq. 25) we obtain \( p_{13}(\omega) \) as

\[
p_{13}(\omega) = \frac{p_{11} e_{11} - p_{33} e_{33}}{e_{11} - e_{33}}.
\]

The stiffness \( p_{55} \) is determined by imposing the boundary conditions:

\[
-\tau(u) \nu = g, \quad (x_1, x_3) \in \Gamma^L \cup \Gamma^R \cup \Gamma^\tau,
\]

\[
u = 0, \quad (x_1, x_3) \in \Gamma^B
\]

where

\[
g = \begin{cases} (0, \Delta G), & (x_1, x_3) \in \Gamma^L \cup \Gamma^R \cup \Gamma^\tau, \\ (0, -\Delta G), & (x_1, x_3) \in \Gamma^B,
\end{cases}
\]

The change in shape of the rock sample allows to obtain \( p_{33}(\omega) \) using the relation

\[
tg(\beta(\omega)) = \frac{\Delta G}{p_{33}(\omega)},
\]

\[
(\beta(\omega)),
\]

\[
(\beta(\omega)).
\]
Table 1. Material properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Illite/smectite</th>
<th>Kerogen</th>
<th>Water</th>
<th>Oil</th>
<th>Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_s$ (GPa)</td>
<td>28.4</td>
<td>7</td>
<td>2.25</td>
<td>0.57</td>
<td>0.022</td>
</tr>
<tr>
<td>$K_m$ (GPa)</td>
<td>18</td>
<td>4.3</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\mu_m$ (GPa)</td>
<td>12.5</td>
<td>1.3</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\rho_s$ (g cm$^{-3}$)</td>
<td>2.7</td>
<td>1.4</td>
<td>1</td>
<td>0.7</td>
<td>0.078</td>
</tr>
<tr>
<td>$\phi$ (per cent)</td>
<td>10</td>
<td>10</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\eta$ (cP)</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>10</td>
<td>0.015</td>
</tr>
<tr>
<td>$\kappa$ (ndarcy)</td>
<td>200</td>
<td>200</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$S_w$ (per cent)</td>
<td>99</td>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$S_o$ (per cent)</td>
<td>0</td>
<td>90</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$S_g$ (per cent)</td>
<td>1</td>
<td>10</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Figure 2. Polar representation of the energy velocities of the $qP$ and $qSV$ waves for the FE 2PBM and analytical SPBM models at 50 Hz. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer (relation 9–1). The results of the analytical model are obtained as effective single-phase fluids.

Figure 3. Polar representation of the dissipation factors of the $qP$ waves for the FE 2PBM and analytical SPBM models at 50 Hz. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer. The results of the analytical model are obtained as effective single-phase fluids.

Figure 4. Polar representation of the dissipation factors of the $qSV$ waves for the FE 2PBM and analytical SPBM models at 50 Hz. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer. The results of the analytical model are obtained as effective single-phase fluids.

Figure 5. Polar representation of the energy velocities of the $SH$ waves for the FE 2PBM and analytical SPBM models at 50 Hz. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer. The results of the analytical model are obtained as effective single-phase fluids.
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Figure 6. Velocity of waves parallel (‘11’ waves) and normal (‘33’ waves) to the layering plane as a function of frequency. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer.

Figure 7. Dissipation factor of waves parallel (‘11’ waves) and normal (‘33’ waves) to the layering plane as a function of frequency. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer.

where $\beta(\omega)$ is the departure angle between the original positions of the lateral boundaries and those after applying the shear stresses, which can be determined by measuring the average horizontal displacement at $\Gamma^T$ (Santos & Carcione 2015).

Finally, the stiffness $p_{\delta\delta}$ is obtained using the boundary conditions:

$$-\mathbf{\sigma}(\mathbf{u})\mathbf{r} = \mathbf{g}_2, \quad (x_1, x_2) \in \Gamma^B \cup \Gamma^R \cup \Gamma^T, \quad (32)$$

$$\mathbf{u}_s = 0, \quad (x_1, x_2) \in \Gamma^L, \quad (33)$$

where

$$\mathbf{g}_2 = \begin{cases} (\Delta G, 0), & (x_1, x_2) \in \Gamma^B, \\ -\Delta G, 0), & (x_1, x_2) \in \Gamma^T, \\ 0, -\Delta G), & (x_1, x_2) \in \Gamma^R. \end{cases}$$

Then, we proceed as indicated for $p_{\delta\delta}(\omega)$.

The approximate solution of these five BVP was computed using an FE procedure. On each cell of the FE partition of the computational domain, we used bilinear functions to approximate each component of the solid displacement vector, while for the non-wetting and wetting fluid displacements we used a closed subspace of the vector part of the Raviart–Thomas–Nedelec space of zero order (Raviart & Thomas 1975). See Santos & Carcione (2015) and Santos & Gauzellino (2017) for details on the description of
Figure 8. Polar representation of the energy velocities of the $qP$ and $qSV$ waves for the FE 2PBM model at 50 Hz as a function of gas saturation in kerogen layers. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer.

Figure 9. Polar representation of the dissipation factors of the $qP$ waves for the FE 2PBM model at 50 Hz as a function of gas saturation in kerogen layers. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer.

Figure 10. Polar representation of dissipation factors of the $qSV$ waves for the FE 2PBM model at 50 Hz as a function of gas saturation in kerogen layers. The medium consists of a sequence of nine water–gas saturated illite–smectite layers and one oil–gas saturated kerogen layer.

Figure 11. Polar representation of the energy velocities of the $qP$ waves for the FE 2PBM and SPBM models at 50 Hz as a function of kerogen concentration. The medium consists of a sequence of eight (seven) water–gas saturated illite–smectite layers and two (three) oil–gas saturated kerogen layers. $S_g = 10$ per cent in the illite–smectite and kerogen layers.

Figure 12. Polar representation of the dissipation factors for $qP$, $qSV$ and $SH$ waves are obtained as in appendices A and B of Santos & Carcione (2015).

The FE experiments consider square periodic layered samples $\Omega$ of side length 0.09 cm with six periods of illite–smectite and kerogen layers (see Fig. 1), discretized using a $60 \times 60$ uniform mesh, that is, $\Omega = \bigcup_{j=1}^{6} \Omega_j$. The material properties are given in Table 1.
4.1 Numerical simulations

The experiments consider a square sample of side length 0.09 cm with an alternating sequence of 0.0135 cm of illite–smectite and 0.0015 cm of kerogen layers, each layer saturated by a two-phase fluid. In the illite–smectite layers, the wetting and non-wetting phases are water and gas, with residual saturations $S_w = 4.5$ per cent and $S_g = 0$, respectively, and gas saturation is $S_g = 1$ per cent. In the kerogen layers, the wetting and non-wetting phases are oil and gas, with residual saturations $S_w = S_g = 4.5$ per cent and $S_g = 0$, respectively, and gas saturation is $S_g = 10$ per cent.

The experiments compare energy velocities (Santos et al. 2014) and dissipation factors of $qP$, $qS\!V$ and $SH$ waves computed using the 2PBM, when the sample is saturated by a two-phase fluid mixture, with the velocities obtained with the analytical solution using the SPBM model as in Krzikalla & Müller (2011). The properties of the single-phase fluids are determined by weighting those of the water–gas and the oil–gas mixtures with the corresponding saturations. The effective single-phase fluid viscosity $\eta^{(\text{eff})}$ and density $\rho^{(\text{eff})}$ are obtained as arithmetic averages of those of the water–gas or oil–gas viscosities, while the effective bulk modulus $K^{(\text{eff})}$ was determined using a Reuss average of the water–gas or oil–gas bulk moduli:

$$\eta^{(\text{eff})} = \eta_w S_w + \eta_g S_g,$$

$$\rho^{(\text{eff})} = \rho_w S_w + \rho_g S_g,$$

$$\frac{1}{K^{(\text{eff})}} = \frac{S_w}{K_w} + \frac{S_g}{K_g}.$$

Small differences between energy velocities of the $qP$ and $qS\!V$ waves at 50 Hz for the FE 2PBM and analytical models can be observed due to capillary pressure and relative permeability effects present in the 2PBM (Fig. 2). The dissipation factors of the $qP$ and $qS\!V$ waves are much higher for the 2PBM than for the SPBM (Figs 3 and 4). Furthermore, attenuation is higher at angles between 60 and 90 deg for $qP$ waves and at angles between 30 and 60 deg for $qS\!V$ waves.

The higher attenuation predicted by the 2PBM model is due to the combined effects of relative permeability and capillary pressure. Relative permeabilities define the dissipation function in the Lagrangian formulation of the 2PBM (Santos et al. 1990a), and they represent the interaction between the two fluid phases as they move within the pore space. To quantify this effect, we have computed the $L^2$ norm of the horizontal and vertical displacements of both fluid phases for the $p_{11}$ and $p_{13}$ tests. For the $p_{11}$ test, the $L^2$ norm of the horizontal displacement of the non-wetting phase is higher than that of the wetting phase.
of the wetting phase, while for the $p_{33}$ experiment this behaviour was observed for the vertical displacements. The same behaviour of the displacements of the two fluid phases was observed in all the experiments performed in this section. These relative motions between the two fluid phases induce energy losses not present in single-phase fluids.

The energy velocities of $SH$ waves are not affected by the relative permeability and capillary pressure (Fig. 5). This behaviour is explained by the fact that $SH$ waves are uncoupled of the $qP$ waves and the shear experiment associated with the $SH$ waves does not induce changes in fluid pressure.

Next, we analyse the behaviour of the phase velocities and dissipation factors of waves as they travel parallel and normal to the layers as function of frequency. In particular, this study allows to identify the possible existence and location of attenuation peaks. The following experiment analyses the behaviour of waves as a function of frequency in the range 1 Hz–1 kHz. Fig. 6 displays velocities of waves parallel (‘11’ waves) and normal (‘33’ waves) to the layering plane, while Fig. 7 shows the corresponding dissipation factors. Velocities increase with frequency. Furthermore, ‘11’ waves exhibit higher phase velocities than ‘33’ waves. Dissipation

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**Figure 15.** Patchy gas saturation distribution in the kerogen layers. The white regions correspond to $S_g = 30$ per cent, the black regions correspond to $S_g = 1$ per cent. Overall gas saturation in the kerogen layers is 10 per cent. The sample is a square of side length 0.09 cm.

**Figure 16.** Polar representation of energy velocities of the $qP$ and $qSV$ waves for the FE 2PBM and FE SPBM models at 50 Hz. Pathy gas-oil distribution in the kerogen layers with overall gas saturation $S_g = 10$ per cent. The medium consists of a sequence of six water–gas saturated illite–smectite layers and four oil–gas saturated kerogen layers (Kerogen concentration is 40 per cent). $S_g = 1$ per cent in the illite–smectite layers.

**Figure 17.** Polar representation of dissipation factors of the $qP$ and $qSV$ waves for the FE 2PBM and FE SPBM models at 50 Hz. Pathy gas-oil distribution in the kerogen layers with overall gas saturation $S_g = 10$ per cent. The medium consists of a sequence of six water–gas saturated illite–smectite layers and four oil–gas saturated kerogen layer (Kerogen concentration is 40 per cent). $S_g = 1$ per cent in the illite–smectite layers.
factors are frequency dependent with attenuation peaks of associated quality factors $Q = 50$ at about 50 Hz for ‘11’ waves and $Q = 67$ at about 60 Hz for ‘33’ waves.

In reservoir rocks saturated by two-phase water–gas or oil–gas mixtures, a certain percentage of immobile water or oil (the wetting phases) always exists, indicated by the residual wetting saturation $S_w$. Thus, in the analysis that follows, the residual saturations are $S_w = 0$, $S_o = 10$ per cent.

### 4.2 Sensitivity to gas saturation in kerogen layers

To analyse changes in energy velocities and dissipation factors due to variations of gas saturation in the kerogen layers, we consider the same sample of the validation experiments but $S_k = 10$ and 30 per cent in the kerogen layers. The energy velocities of the $qP$ and $qSV$ waves for the 2PBM are not sensitive to changes in gas saturation in the kerogen layers (Fig. 8). The corresponding values of the energy velocities for the SPBM are not shown due to their small differences with those of the 2PBM.

The dissipation factors of the $qP$ and $qSV$ waves as a function of the propagation angle at 50 Hz are shown in Figs 9 and 10, respectively. For $qP$ waves, attenuation is higher for waves travelling normal to the layering plane, and higher for $S_k = 10$ per cent than for $S_k = 30$ per cent. The attenuation predicted by the SPBM model exhibits a similar behaviour but with much lower values.

For $qSV$ waves, attenuation is stronger for angles between 30 and 60 deg, and higher for $S_k = 30$ per cent than for $S_k = 10$ per cent. Attenuation values obtained using the SPBM model are negligible and are shown as a point at the origin. As in the previous example, relative permeabilities are responsible for the high attenuation predicted by the 2PBM model.

### 4.3 Sensitivity to kerogen concentration

Here, we analyse changes in the energy velocities and dissipation factors of $qP$ and $qSV$ waves due to variations in the kerogen concentration. We consider the same sample of the validation experiments with six periods of 0.012 cm of illite–smectite and 0.003 cm of kerogen (20 per cent kerogen) and six periods of 0.0105 cm of illite–smectite and 0.0045 cm of kerogen (30 per cent kerogen). As expected, lower velocity corresponds to higher kerogen content (Figs 11 and 12). Furthermore, much higher dissipation factors are observed for the 2PBM model than for the SPBM model, and a completely different anisotropic behaviour (Figs 13 and 14). These results indicate that the SPBM model is not reliable for predicting attenuation in multiphase saturated porous rocks.

### 4.4 Sensitivity to patchy saturation

Finally, we analyse the effect of patchy gas–oil saturation in the kerogen layers for the case of 40 per cent kerogen concentration. Patchy-saturation patterns produce strong mesoscopic-loss effects at the seismic frequency band, as shown by White et al. (1975).

To generate patchy gas–oil distribution in the kerogen layers, we proceed as follows. The first step to generate a patchy fluid distribution is to assign to each subdomain $\Omega_j$ of the partition of the domain $\Omega$, a pseudo-random number using a generator with uniform distribution. This random field is Fourier transformed to the spatial wavenumber domain and its amplitude spectrum is multiplied by the von Karman spectral density given by (Frankel & Clayton 1986; Santos et al. 2005)

$$S_k(k_x, k_z) = S_0(1 + k^2(\text{CL})^2)^{-1/2},$$

where $k = \sqrt{(k_x^2 + k_z^2)}$ is the radial wavenumber, $N_e$ is the Euclidean dimension, $\text{CL}$ the correlation length, $H$ is a self-similarity coefficient ($0 < H < 1$) and $S_0$ is a normalization constant. Eq. (34) corresponds to a fractal process of dimension $D = N_e + 1 - H$ at scales smaller than $\text{CL}$. The resulting fractal spectrum is then transformed back to the spatial domain, obtaining a microheterogeneous fractal gas saturation model $S_g^{(j)}$.  

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**Figure 18.** Absolute value of the fluid pressure for the 2PBM model at 50 Hz, patchy gas–oil saturation with 10 per cent overall patchy gas saturation in the kerogen layers. The medium consists of a sequence of six water–gas saturated illite–smectite layers and four oil–gas saturated kerogen layer (Kerogen concentration is 40 per cent). $S_w = 1$ per cent in the illite–smectite layers.
Next, to assign to each cell $\Omega$ either $S_g = 1$ per cent or $S_g = 30$ per cent, a threshold value $S_b^*$ is chosen so that for each subdomain $\Omega$, where $S_b^{(i)} \leq S_b^*$ it is assumed that such subdomain has $S_g = 1$ per cent, while if $S_b^{(i)} > S_b^*$, $S_g = 30$ per cent in $\Omega_i$. In this way, a multiscale binary gas–oil patchy-saturation model is constructed and an overall brine saturation $S_b^*$ is obtained. In the examples, the fractal dimension is $D = 2.3$ and the correlation length is 1.67 per cent of the side length of the sample. Residual saturations are $S_w = 10$ per cent and $S_g = 0$. Saturation in the illite–smectite layers is chosen to be uniform with gas saturation $S_g = 1$ per cent.

Fig. 15 displays the patchy gas–oil distribution in the kerogen layers. The white regions correspond to $S_g = 30$ per cent, and the black regions correspond to $S_g = 1$ per cent. Figs 16 and 17 show the energy velocities and dissipation factors of $q_P$ and $q_S$ waves at 50 Hz for the SPBM and 2PBM models and patchy gas–oil saturation in the kerogen layers for overall gas saturation 10 per cent. The results of the SPBM were obtained using the FE harmonic experiments as in Picotti et al. (2010) with the effective single-phase fluid properties determined as in eq. (34).

Energy velocities of $q_P$ and $q_S$ waves are very similar for both models (Fig. 16). On the other hand, the attenuation of the $q_P$ waves is almost isotropic for the SPBM model, while the 2PBM model exhibits much higher attenuation and strong anisotropy (Fig. 17). Furthermore, $q_S$ attenuation is strong for angles between 30 and 60 deg and higher for the 2PBM model than for the SPBM model.

Fig. 18 shows the absolute value of the total fluid pressure distribution $\mathcal{P}$ at 50 Hz, defined as $\mathcal{P} = \mathcal{T}_g + \mathcal{T}_w$, with $\mathcal{T}_g$ and $\mathcal{T}_w$ being the generalized forces in eqs (3) and (4), respectively. It is seen that pressure gradients are the highest at the gas–oil interfaces. This illustrates the WIFF mechanism.

5 CONCLUSIONS

We have shown that in porous rocks saturated with two-phase fluids, the presence of capillary forces (interfacial tension) and the relative permeabilities, significantly affect the attenuation of $q_P$ and $q_S$ waves. We considered shales composed of illite–smectite layers saturated with water and gas, and kerogen layers saturated with oil and gas. Quasi-static numerical experiments performed with an FE procedure allowed us to compute the energy velocities and dissipation factors due to WIFF. The higher attenuation and strong Q anisotropy predicted by the 2PBM are due to the combined effects of relative permeability and capillary pressure. Relative permeabilities define the dissipation function in the Lagrangian formulation, representing the interaction between the two fluid phases as they move within the pore space. These relative motions induce energy losses not present in rock saturated with single-phase or effective fluids.

REFERENCES


