

# **Attenuation and Dispersion of Elastic Waves in Porous Rocks:**

**Mechanisms and Models**

Boris Gurevich and José M. Carcione

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# Preface

Wave propagation in all fluid and solid matter is always accompanied by attenuation and dispersion in a broad range of frequencies and scales from free oscillations of the entire Earth to ultrasound in small rock samples (Aki and Richards, 2002). Attenuation is the exponential decay of wave amplitude with propagation distance; dispersion is a variation of propagation velocity with frequency. Attenuation and dispersion can be caused by a variety of physical phenomena that can be divided broadly into two main classes: elastic processes, where the total energy of the wavefield is conserved (scattering of the waves by randomly distributed inhomogeneities), and inelastic dissipation (internal friction), where wave energy is converted into heat (see, e.g., Sato et al., 2012). In fluid-saturated porous media, the most common cause of internal friction is wave-induced motion of the fluid relative to the solid frame. Often, this phenomenon is termed wave-induced fluid flow (WIFF) or, perhaps more accurately, wave-induced fluid-pressure relaxation. Understanding of the fluid-related dissipation, combined with improved measurements of attenuation and/or dispersion from experimental data, can be useful for a number of applications, such as estimation of hydraulic properties of rocks. Dissipation-related seismic attributes already are employed in seismic interpretation and reservoir characterization, but to date their use has been mostly empirical and qualitative (see, e.g., Castagna et al., 2003; Rapoport et al., 2004; Hermana et al., 2014; Li et al., 2016). Theoretical models of frequency-dependent attenuation and dispersion may help develop *quantitative* attributes, which can be calibrated against well logs and laboratory measurements. Such theoretical models are the subject of this book.

The first description of attenuation and dispersion due to WIFF probably should be credited to prominent Russian physicist Jacob Frenkel (Frenkel, 1944). Subsequently, his theoretical analysis has been expanded in a series of papers by Belgian-American physicist Maurice A. Biot (Biot, 1956a,b, 1962a,b). The Biot theory of dynamic poroelasticity is based on his earlier analysis (Biot, 1941) of quasi-static deformation of fluid-saturated porous media, which in turn is an extension to porous media with an elastic frame of the concepts of soil mechanics developed by Karl von Terzaghi and his followers in 1920s and 1930s (see, e.g., Terzaghi, 1943). The dispersion/attenuation mechanism described by the Biot theory is caused by pressure relaxation between peaks and troughs of a passing wave, which, in turn, occurs due to the density differences between the solid frame and the pore fluid. When the fluid is inviscid, application of the same pressure to the medium results

in different particle velocities in the solid and fluid, and longitudinal waves propagating in the matrix and fluid (known as fast and slow waves, respectively) with no dissipation or dispersion. In the fast wave, the fluid and solid motion is almost in-phase, and in the slow wave out-of-phase. When the fluid is viscous, attenuation is very high in the slow wave (especially at low frequencies) and relatively low in the fast wave – but nonzero due to some small motion of the fluid relative to the solid.

To visualize this phenomenon, consider a solid pipe filled with a viscous fluid being shaken back and forth in the axial direction. If the fluid is inviscid, it should have remained stationary. But if the fluid is very viscous, it will be dragged along by viscous forces, but it will lag behind the solid pipe, creating relative motion and hence energy dissipation. The same phenomenon occurs in a porous medium, although the tortuous pore space results in additional inertial coupling between the solid and fluid motion.

This dissipation mechanism often is called global or macroscopic flow because it occurs on the scale of a wavelength, which, at frequencies below 1 MHz, is much larger than an individual pore size. The slow compressional wave predicted by the Biot theory is observed experimentally by Plona (1980) but most relevant for many applications is attenuation and dispersion of the fast compressional wave.

The development of the Biot theory of poroelasticity in the 1950s and 1960s provided a solid understanding as to how the presence of fluids in the pore space of a rock can cause attenuation and dispersion. This was exciting for many applications, such as petroleum geophysics, as attenuation and dispersion potentially could become useful attributes for seismic exploration (it is worth noting that Biot developed the poroelasticity theory while working for the oil giant Shell). However, it soon became apparent that the attenuation and dispersion predicted by the poroelasticity theory was prominent only at frequencies of 100 kHz or higher, and negligibly small at seismic (10–100 Hz) and sonic (100–10,000 Hz) frequencies. Furthermore, progress in laboratory measurements and field observations in the 1970s, both in ocean acoustics and borehole seismic methods, showed that measured attenuation of seismic and acoustic waves at frequencies between 10 Hz and 10 kHz was one to two orders of magnitude higher than the poroelasticity predictions.

This discrepancy has prompted a quest to find other mechanisms of dissipation. One approach is to include these mechanisms as an additional attenuation in the rock frame, making it viscoelastic. Apparently, the first to propose such an approach is Stoll and Bryan (1970). Over the decades, this approach has experienced various extensions and refinements (see, e.g., Carcione, 2022). While this approach cannot shed light on the physics of attenuation and dispersion, sometimes it is useful in modeling the observations.

Furthermore, also realized in the 1970s is the idea that the global flow alone does not account for all of the WIFF effects in porous rocks. In particular, if a rock has heterogeneities smaller than the wavelength, a passing wave will create pressure gradients on a length scale of these heterogeneities. This, in turn, will cause fluid flow relative to the solid, and hence attenuation and dispersion. Depending on the actual scale of the heterogeneities, we can distinguish between mesoscopic flow and local flow. Mesoscopic flow is caused by heterogeneities having a length scale is much smaller than the wavelength

but much larger than the typical pore size. When a porous medium with mesoscopic heterogeneities is compressed by a passing wave, the fluid will tend to flow (squeeze out) from more compliant into stiffer regions, and the flow will cause dissipation due to viscous friction.

Local flow, also known as squirt flow, is similar in nature but occurs on the pore scale. If pores have irregular shapes and orientations (which is almost always the case), they have different compliances. Under compression, the more compliant pores will deform to a greater extent and will squeeze the fluid into stiffer pores. The physical natures of mesoscopic and squirt flow are quite similar, but the geometry and mathematical formulations often are very different.

This book aims to give the reader a detailed description of all of the dissipation mechanisms caused by fluid-pressure relaxation (on macroscopic, mesoscopic, and pore scales). For each mechanism, we strive to describe its physics, theoretical models, and experimental evidence for the physical mechanisms and models. The choice of models reflects the authors' interests and is based in large part on the work of the authors, their research students, and co-workers. The focus of the book is on theory; experimental examples are chosen to illustrate the physics and theoretical models and are far from comprehensive. A complete analysis of experimental data on attenuation and dispersion in porous materials will perhaps require another book.

The structure of the book is as follows. **Chapter 1** introduces the concept of internal friction and gives its theoretical description through the theory of viscoelasticity, which underpins the concepts of dispersion and attenuation. The chapter introduces the theoretical description of viscoelastic media, discusses classical rheological models, and defines the key quantities governing mechanical deformation and wave propagation in these media, such as moduli, compressional and shear wave velocities, and attenuation and dissipation factors. Also introduced here are mathematical concepts of complex-valued moduli and velocities, which are useful for a mathematical description of waves in dissipative media. The theory of viscoelasticity is a classical subject covered in many books; hence, this chapter is introductory in nature and focuses on the main concepts and definitions necessary for subsequent chapters.

The Biot theory of dynamic poroelasticity is the subject of **Chapter 2**. The theory is well established and covered in a number of books, e.g., Nikolaevskiy et al. (1970), Bourbié et al. (1987), Allard and Atalla (2009), Coussy (2011), Carcione (2022), and Cheng (2016). The purpose of this chapter is three-fold: to give a detailed description of the global-flow mechanism described by this theory, to illustrate this mechanism with experimental data, and to present field equations and constitutive equations, which are employed in subsequent chapters to develop mathematical models of other attenuation mechanisms that occur in porous media with strong spatial heterogeneity. Note that, strictly speaking, unique among the mechanisms discussed in this book, the global-flow dissipation described by the Biot theory cannot be described within the framework of viscoelasticity (Chapter 1). Indeed, a mathematical description of global-flow dissipation requires the inertial terms in the equations of poroelasticity, and thus cannot be described

by a quasi-static viscoelastic relaxation.

Attenuation and dispersion caused by fluid-pressure relaxation on the mesoscale is covered in **Chapters 3 to 5**. The concept of mesoscopic pressure relaxation has been developed in two papers by J. E. White and co-workers (White et al., 1975; White, 1975). In many natural and artificial porous media, mesoscopic flow can occur on a wide range of scales from the largest pore size to the smallest wavelength, and thus can cause attenuation in a broad range of frequencies. Therefore, mesoscopic flow increasingly is believed to be a key mechanism of fluid-related attenuation in the seismic exploration frequency band (Pride et al., 2004). To cause significant attenuation and dispersion, the degree of heterogeneity, that is, the contrast between elastic properties of different portions of the rock, must be relatively large. There are a number of scenarios where this is manifestly the case. The two most obvious scenarios, which have received substantial attention in recent years, are partially saturated media and fractured porous media. Chapter 3 introduces the concept of mesoscopic relaxation and gives a systematic description of attenuation and dispersion in media with both periodic and randomly distributed inhomogeneities. The models described in this chapter are generic, in a sense that the medium can be spatially inhomogeneous in any of its properties, such as porosity, frame moduli, or pore fluid modulus.

**Chapter 4** focuses on a particular case of mesoscopic relaxation where only the pore-fluid properties vary on the mesoscale while the solid frame properties are all spatially uniform. If two immiscible pore fluids with substantially different fluid bulk moduli (such as liquid and gas) occupy mesoscopic-scale clusters of pores, significant wave attenuation and dispersion may occur due to the buildup of pressure gradients between these clusters. The complete pressure equilibration between the two fluids requires the frequency to be sufficiently low so that the characteristic length of fluid-pressure diffusion in the pore space is large compared to the largest spatial scale of fluid mixing. If the frequency is higher, the pressure in the two fluids will not have sufficient time to equilibrate within a half-period of the wave, resulting in a higher bulk modulus and wave velocity. Hence, the presence of two fluids in the pores causes significant dispersion and attenuation of elastic waves, which is related to the relaxation of pore-fluid pressures. The frequency dependence of wave velocity and attenuation in a partially saturated medium is controlled by the size, shape, and spatial distribution of fluid pockets, and the permeability and elastic moduli of the solid frame, as well as the properties of the two fluids. Compared to general inhomogeneous porous media, partially saturated media with a uniform frame allow for more tractable theoretical analysis and, no less importantly, for controlled experiment (i.e., it is easier to control the fluid saturation in a porous medium than to control its porosity). Partially saturated media are also of significant practical importance, particularly for geophysical exploration and monitoring of hydrocarbon reservoirs, or for monitoring CO<sub>2</sub> injection into porous rocks.

A strong degree of heterogeneity is also typical of fractured reservoirs such as tight sands and carbonates, where compliant mesoscopic fractures embedded in a porous rock mass play a crucial role as flow conduits (Nelson, 2001). During the compression wave cycle,

there is WIFF from compliant fractures into the porous background medium and vice versa during the dilatational wave cycle. If the fractures are aligned in space, then fluid-pressure relaxation between pores and fractures will cause frequency-dependent anisotropy. Seismic characterization of fracture sets is an important aspect of reservoir characterization, and therefore attenuation due to wave-induced flow and associated frequency-dependent anisotropy have attracted considerable research interest. These phenomena are covered in **Chapter 5**.

Pore-scale stress relaxation is the subject of **Chapters 6 and 7**. The concept of local (or squirt) flow, corresponding to pore-scale relaxation of fluid pressure between various pores and cracks of different shapes and orientations, is foreshadowed by Biot (1962b) and then developed by Mavko and Nur (1975) and O'Connell and Budiansky (1977). **Chapter 6** is focused on pore-scale relaxation of pore-fluid pressure between more compliant voids (cracks, grain-to-grain contacts) and relatively stiff pores. When the rock is compressed, much greater pressure builds up in compliant than stiff pores, resulting in a fluid-pressure gradient, local fluid flow, and dissipation. Similarly to the case in mesoscopic relaxation, when the frequency is low, the fluid pressure has sufficient time to equalize within one half-period of the wave, and hence the compliant pores remain compliant. Conversely, at high frequencies, there is insufficient time for the pressure to equalize, and hence the pores that are compliant in the dry state become much stiffer. Therefore, materials with a binary pore structure exhibit significant moduli and velocity dispersion. At the same time, the presence of compliant pores is responsible for the dependence of elastic wave velocities on static pressure. Thus, substantial reduction of the pore compliance at high (e.g., ultrasonic) frequencies results in much weaker pressure dependence. This effect can be demonstrated by a much stronger dependence of the bulk modulus on the static pressure at seismic than ultrasonic frequencies.

Attenuation may be even stronger in porous media whose pores are filled with viscoelastic substances such as heavy oil/bitumen. At low frequencies (and/or high temperatures), the pore fill is in a liquid state, the fluid pressure is equalized between compliant and stiff pores, and the rock is relatively soft. But at high frequencies (or low temperatures), the pore fill is near-solid, and obviously cannot flow, making previously compliant pores very stiff and causing strong dispersion. Analysis of elastic and viscoelastic properties of porous media filled with solid and viscoelastic substances is the subject of **Chapter 7**. Although a porous medium whose pores contain an elastic substance is itself elastic and does not exhibit any dissipation, its effective moduli can be described with the same approach as employed to describe the squirt-flow relaxation (Chapter 6). Moreover, using the general elastic/viscoelastic correspondence principle (Chapter 1), the result for a solid-filled medium can be extended to porous media saturated with viscoelastic substances, which can exhibit very large attenuation and dispersion.

Finally, **Chapter 8** shows that complex-valued moduli (which describe attenuation and dispersion) of fluid-saturated porous media must lie within rigorous bounds. These bounds are a particular case of more general bounds derived by Gibiansky and Milton (1993), Milton and Berryman (1997), and Gibiansky et al. (1999) for two-phase mixtures

of viscoelastic substances. The derivation presented in Chapter 8 shows that, for fluid-saturated porous media, the bounds for both bulk and shear moduli are semicircles in the complex plane, with the diameter being the segment of the real axis between the nominal Hashin–Shtrikman bounds for a porous medium saturated with an inviscid fluid. We also show that these bounds are independent of frequency and realizable – that is, for every modulus value within the bounding region, there exists a geometry of the porous material for which the modulus equals this value.

The book also contains two appendices. **Appendix A** gives a condensed summary of effective medium theories for elastic and viscoelastic composites. We cover both deterministic and statistical models, and both static and dynamic effective properties. The material is presented in a handbook style and is referenced throughout the book. **Appendix B** gives a detailed summary of the main methods for measuring attenuation and dispersion in the laboratory. A basic knowledge of these methods is necessary for understanding experimental data presented in various chapters.

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